

Science
S0121
Statistics Paper II

[Time: Three Hours]

[Marks:100]

Please check whether you have got the right question paper.

- N.B: 1. All questions are compulsory.
2. Use of calculator is allowed.

- Q. 1 (a) Correct the following if necessary. Justify each (correct or wrong) statement. (10)**
- (1) For complementary event A $P(\bar{A}) = P(A) - 1$. (02)
- (2) Two events A and B are independent if $P(A \cup B) = P(A) * P(B)$. (02)
- (3) Number of messages on your mobile in a day is an example of continuous random variable. (02)
- (4) Binomial distribution with parameters 'n' and 'p' tends to Poisson distribution if both 'n' and 'p' tends to infinity. (02)
- (5) Random variable X and Y are independent if $COV(X, Y) = 0$. (02)
- (b) Explain the terms giving suitable examples. (10)**
- (1) Mutually Exclusive events. (02)
- (2) Equally likely events. (02)
- (3) Random experiment. (02)
- (4) Discrete random variable (r.v). (02)
- (5) Bernoulli trial. (02)
- Q. 2 Attempt any TWO sub-questions. (10)**
- (1) I) State the addition theorem for three events. (10)
II) Prove that $P(A \cup B) \leq P(A) + P(B)$.
III) A bag contains 5 white and 4 black balls. A ball is drawn from this bag and is replaced and then second draw of a ball is made. What is the probability that two balls are of the different colors.
- (2) I) If A and B are any two events associated with sample space S of an experiment, then prove that $P(A \cap B) = P(A) * P(B/A)$ (10)
What will be modification in the above expression if A and B are independent.
II) A speak truth in 75% cases, B in 80% cases. In what percentage of cases they are likely to contradict each other.
- (3) I) State and prove Bayes' theorem on probability. (10)
II) If A and B are two independent events show that i) A and \bar{B} ii) \bar{A} and B are also independent.
- (4) I) Explain the concept of pair wise and mutual independence for three events. (10)
Does pairwise independence implies mutual independence? Justify.
II) Give statistical definition of probability and state its limitations.

Q. 3 Attempt any TWO sub-questions.

- (1) Show that $V(aX+bY) = a^2V(X) + b^2V(Y) + 2ab\text{COV}(X, Y)$. Where a, b are non zero constants, Hence write expression for i) $a=1, b=-1$ ii) X, Y are independent iii) $\text{Corr}(X, Y)=0.6$. (10)
- (2) Define central and raw moments. Obtain the relationship between first four central moments in terms of first four raw moments. (10)
- (3) I) State and prove addition theorem on Expectation for two random variables. (10)
II) Prove that (i) $E(X+D) = E(X)+D$ D is constants.
(ii) $V(CX) = C^2 V(X)$ C is constant.
- (4) I) Explain following terms i) Two dimensional discrete r.v ii) joint probability mass function (p. m. f) of r. v. s X, Y iii) marginal p. m. f of X iv) conditional p.m.f. of Y given $X=x$ v) Independence of X, Y . (10)
II) State properties of Cumulative Distribution Function (c. d. f) of discrete r.v.

Q. 4 Attempt any TWO sub-questions.

- (1) For Poisson distribution show that its mean is equal to variance. (10)
- (2) I) A biased coin is tossed 6 times. The probability of heads on any toss is 0.3. Let X denote the number of heads that come up. What is $P\{X=2\}$? Find its mean, S.D and mode. (10)
II) Derive the recurrence relation for the calculation of probabilities of Binomial distribution with parameters n and p .
- (3) I) Under conditions to be stated show that binomial distribution tends to Poisson distribution. (10)
II) Derive the mean and standard deviation of Hypergeometric distribution.
- (4) Define Discrete Uniform distribution. Give real life example. For X assuming discrete uniform over $0, 1, \dots, n-1$. Derive its mean and variance. (10)

Q.5 Attempt any FOUR sub-questions.

- (1) State and prove the addition theorem on probability for any two events. (05)
- (2) Mean and variance of Binomial distribution are 5 and $5/6$ respectively. Find $P(X=1)$ (05)
- (3) Define i) Sample space ii) an event iii) probability mass function of discrete r.v iv) mathematical expectation of a random variable X , v) Conditional Probability. (05)
- (4) X and Y are stochastically independent random variables with means 7 and 4 and variances 9 and 4 respectively. Find $E(3X+2)$ and $V(X+Y)$ (05)
- (5) Derive recurrence relation for the probabilities of Poisson distribution. (05)
- (6) A ticket is drawn from a box containing tickets numbers 1, 2, 3,.....10. Find i) the mean number shown on a ticket. ii) P [Number shown is odd] (05)
- (7) Suppose that a researcher goes to a college with 200 faculty, 40 of which have blood type O-negative. She obtains a simple random sample without replacement of size 20. What is the probability that a sample contains. (05)
i) Exactly 2 faculty ii) At most 2 faculty that have blood type O-negative. Determine the mean and standard deviation of the number of selected faculty that will have blood type O-negative.