Q. P. Code: 50944

(Time: 03 hours) **[Total Marks: 60]**

N.B.

i. Attempt any two questions from question numbers 1, 2, 3, and any two questions from question numbers 4, 5, and 6.
ii. Figures to the right indicate full marks.
iii. Simple non-programmable calculator is allowed.

1 a. A trial results in any one of k outcomes $A_1, A_2, \ldots, A_k$ with respective probability $p_1, p_2, \ldots, p_k$ where $\sum_{i=1}^{k} p_i = 1$. Suppose the trial is repeated until A_k is observed exactly r times where r = 1,2,... Obtain: (06)

i. joint probability distribution of $(X_1, X_2, \ldots, X_{k-1})$ where X_i: number of occurrences of $A_i \ \forall\ i = 1,2,\ldots,k$.
ii. probability distribution of $\sum_{i=1}^{(k-1)} X_i$.
iii. $Cov(X_1, X_2)$.

b. A population consists of N animals of which M have been marked. Let Y denote the sample size required to recapture exactly k marked animals. Assuming the sampling scheme to be without replacement, derive the probability distribution of Y, its mean and variance. (05)

c. Show that 'variance of X equal to zero' is a necessary and sufficient condition for a random variable X to be degenerate. (04)

2 a. If X follows Normal distribution with parameters (μ, σ^2), obtain the moment generating function of X. Hence derive variance of e^X. (05)

b. Suppose X has pdf, (05)

$f(x) = e^{-(x-\theta)}, \ x > \theta, \ -\infty < \theta < \infty,$
$= 0$ otherwise.

i. Obtain pdf of Y_1, the first order statistic based on a random sample of size n.
ii. Obtain mean of Y_1.
iii. What is the probability that Y_1 exceeds the median of the distribution of X?

c. If X follows a standard Cauchy distribution, (05)

i. Show that its mean does not exist.
ii. Check for the existence of E(1/X).
iii. Obtain median of the distribution.

CDC44DAE0E2D31E8085E42F2318A8FAC

3 a. Suppose X and Y are independent and identical exponential random variables with parameter θ. Obtain the probability density function of X-Y. Hence or otherwise explain how you will simulate random observations from Laplace (double exponential) distribution. (05)

b. Let X be a random variable defined on [0, 1]. If P ($a < X < b$) depends only on $b - a$, where $0 \leq a \leq b \leq 1$ then show that X is Uniformly distributed over the interval [0, 1]. (05)

c. Let X be a non-negative integer valued random variable satisfying the relation $P(X > m+1 \mid X > m) = P(X \geq 1)$ for any non-negative integer m, then show that X is geometric random variable. (05)

4 a. Define marginal and joint consistency. Prove or disprove "joint consistency implies marginal consistency." (06)

b. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n drawn from the normal distribution with mean θ and variance 1. Obtain Bhattachrya's bound for variance of unbiased estimator of θ^2. (09)

5 a. i. If T_n is consistent estimator of θ then show that any continuous function of T_n say $\varphi(T_n)$ is consistent estimator of $\varphi(\theta)$. (09)

ii. If T_1 and T_2 are two consistent estimators of θ, then state the criterion to decide which is more efficient.

iii. Let $x_1, x_2, \ldots, x_n$ be a random sample from Cauchy distribution with $(\mu, 1)$ Show that sample mean is not consistent for μ. Suggest consistent estimator for it.

b. Derive Cramer-Rao lower bound for variance of any unbiased estimator. (06)

6 a. Define, (04)

i. Equivariant estimator.
ii. Admissible decision rule.
iii. Minimum risk equivariant estimator.
iv. Pitman estimator for location family.

b. Show that Pitman estimator for σ^r is same as Bayes' estimator using Square error loss function where σ is a scale parameter of exponential distribution. (06)

c. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n drawn from $U(0, \theta)$. Obtain Jackknife estimator of θ. (05)

CDC44DAE0E2D31E8085E42F2318A8FAC