

(Time: 2 ½ Hours)

[Total Marks: 75]

- N.B. 1) All questions are compulsory.  
 2) Figures to the right indicate marks.  
 3) Illustrations, in-depth answers and diagrams will be appreciated.  
 4) Mixing of sub-questions is not allowed.

**Q.1 Attempt All (Each of 5 Marks)****(15M)**(a) **Select correct answer from the following:**

- A relation  $R$  on a set  $A$  is said to be \_\_\_\_\_ if  $aRb$ ,  $bRc$  and  $aRc$  for all  $a, b, c \in A$ .  
 a) Reflexive b) Symmetric c) Transitive d) Antisymmetric
- The value of  $P(3, 2) =$  \_\_\_\_\_  
 a) 6 b) 9 c) 8 d) 5
- Two vertices  $V_1$  and  $V_2$  in a graph  $G$  are said to be \_\_\_\_\_ to each other iff they are the end vertices of the same edge  $e$ .  
 a) Adjacent b) Parallel c) Loops d) None
- In \_\_\_\_\_ ways 8 different beads can be arranged to form a necklace  
 a)  $8!$  b)  $7!$  c)  $9!$  d) None
- Diagrammatic representation of a relation  $R$  defined on a set is called \_\_\_\_\_  
 a) Diagraph b) Multigraph c) Hasse diagram d) None

(b) **Fill in the blanks:**

(Indegree, 45, 21, 35, increase, Lattice, POSET, Injective, Surjective)

- A Set together with a partial order relation is called \_\_\_\_\_
- An onto function is called as \_\_\_\_\_ function
- The number of incoming edges on a vertex  $v$  is called \_\_\_\_\_ of a vertex.
- The value of  $C(10, 8) =$  \_\_\_\_\_
- In \_\_\_\_\_ ways 4 questions can be selected form 7 questions.

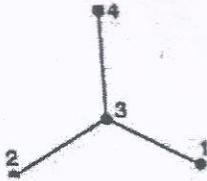
(c) **Define the following.**

- Equivalence Relation
- Recurrence relation
- Simple graph
- Pigeonhole principle
- Pascal identity

**Q.2 Attempt the following (Any THREE)****(15M)**

- (a) If  $f: R \rightarrow R$  is defined by  $f(x) = \frac{(2x-3)}{7}$ , for all  $x \in R$ , then show that  $f$  is a bijective function.
- (b) Define composition function. If  $f$  and  $g$  are two functions from the set of integers to the set of integers defined by  $f(x) = x + 3$  and  $g(x) = x^2$  then find  $f \circ g(x)$  and  $g \circ f(x)$ .

- (c) Define equivalence relation and let  $R = \{(1,1), (1,3), (2,2), (2,4), (3,3), (3,1), (4,4), (4,2)\}$  be the relation defined on  $A = \{1, 2, 3, 4\}$ . Show that  $R$  is an equivalence relation.
- (d) Describe the order pairs in the relation determined by the Hasse diagram of a poset  $(A, \leq)$  on the set  $A = \{1, 2, 3, 4\}$



- (e) Solve the recurrence relation  $a_n = a_{n-1} + 2a_{n-2}$ ,  $n \geq 2$  with initial conditions  $a_0 = 0, a_1 = 1$  Using characteristic root method.
- (f) Explain Tower of Hanoi and solve the puzzle.

**Q. 3 Attempt the following (Any THREE)**

**(15M)**

- (a) Prove that  
 (i)  $C(n,0) = 1$  (ii)  $C(n,1) = n$  (iii)  $C(n,n) = 1$  (iv)  $C(n,r) = C(n,n-r)$
- (b) How many distinguishable permutations of the letters in the word SCIENCE are there?
- (c) Draw a tree diagram to find how many bit strings of length four do not have two consecutive 1's.
- (d) A class is composed of 2 brothers and 6 other boys. In how many ways can all the boys be seated at a round table so that the two brothers are not seated together?
- (e) Let  $L = \{a, ab, a^2\}$  and  $M = \{b^2, aba\}$  be languages over  $A = \{a, b\}$ . Find (i)  $LM$  (ii)  $MM$
- (f) Find the language  $L(G)$  over  $\{a,b,c\}$  generated by the grammar  $G$  with productions  $S \rightarrow aSb, aS \rightarrow Aa, Aab \rightarrow c$ .

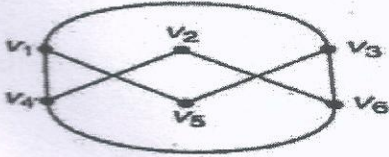
**Q. 4 Attempt the following (Any THREE)**

**(15)**

- (a) Define adjacency matrix and Draw the undirected graph  $G$  corresponding to given adjacency matrix.

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

- (b) What is a planar graph? Draw a planar graph representation of the given graph.



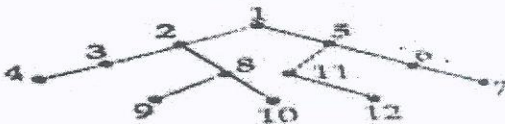
- (c) Explain the operations on graphs also find union and intersection of the given graphs.



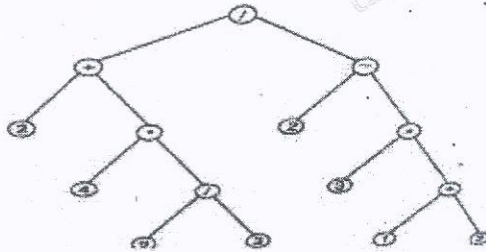
- (d) Use Depth first search algorithm to find a spanning tree for the given graph.



- (e) What is tree traversal and Find preorder, postorder and inorder search for the given tree.



- (f) Determine the value of the expression represented in a binary tree.



**Q. 5 Attempt the following (Any THREE) (15)**

- (a) If  $A = \{1, 2, 3\}$  and  $R$  be relation on  $A$  defined by  $xRy$  such that  $x \leq y$ . Find  $R$  and draw its diagram.
- (b) Using generating function solve the recurrence relation  $a_n = 3a_{n-1} + 2$  with initial condition  $a_0 = 1$ .
- (c) What is the probability that a randomly selected number that is between 100 and 999 (both inclusive) will not contain the digit 7?
- (d) What is a Complete graph. Draw a regular graph with 5 vertices
- (e) Consider the language  $L = \{ab, c\}$  over  $A = \{a, b, c\}$ , Find : a)  $L^3$ ; b)  $L^{-2}$