Con. 3846 & (a) to (c)-13. **CG-2368**

(CG-2368-2371)

(2½ Hours) **[Total Marks : 60**

N.B. : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **marks** for respective **subquestions**.

1. (a) Show that Newton Raphson iteration formula applied to the function $f(x) = x^2 - a$ $(a > 0)$ leads to the iteration formula

$$x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right);\ x_0 > 0$$

for evaluating $\sqrt{a}$. Further show that this sequence has convergence of second order with limit $\sqrt{a}$ (7)

(b) Attempt any **one** question:

(i) Apply Regula Falsi method to find the smallest positive root of the equation $x - e^{-x} = 0$ correct to 3 places of decimal. The root lies between $[0.56, 0.58]$. Perform two iterations. (8)

(ii) For the function $f(x) = x^3 - x - 1$, show that the rearrangement $x(x^2-1) = 1$ leads to a fixed point iteration function $g(x) = \frac{1}{x^2-1}$ and show that the rearrangement $(x-1)(x^2+x+1) - x = 0$ leads to iteration function $g(x) = 1 + \frac{x}{x^2+x+1}$. How do the iterates generated by each of these fixed point functions behave? (In each case compute three iteration starting with $x_0 = 1.5$). (8)

2. (a) If p_k is an approximation of the root p of the polynomial equation $p_n(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$, then show that the next approximation to the root using Birge-Vieta method is $p_{k+1} = p_k - \frac{b_n}{c_{n-1}}$, $k = 0, 1, 2, \ldots$ where b_k satisfies the recursive relation $b_k = a_k + pb_{k-1}$ with $b_0 = a_0$ and c_k satisfies the recurrence relation $c_k = b_k + pc_{k-1}$ with $c_0 = b$. (7)

(b) Attempt any **one** question:

(i) Solve the following system of equations by Cholesky's method. (8)

$$\begin{aligned} 4x_1 + 2x_2 + 14x_3 &= 14 \\ 2x_1 + 17x_2 - 5x_3 &= -101 \\ 14x_1 - 5x_2 + 83x_3 &= 155 \end{aligned}$$

(ii) Determine the sturm's sequence of the polynomial equation (8)

$$f(x) = x^3 - 5x + 1 = 0.$$

3. (a) Describe Jacobi iterative method to reduce the symmetric matrix A to diagonal matrix D for finding the eigenvalues of A. (7)

(b) Attempt any **one** question:

(i) Find all eigenvalues of the matrix $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ at the end of fourth iteration using Rutihauser method. (8)

(ii) Find the smallest eigenvalue of the matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ in magnitude by inverse power method. Take initial approximation as $[1\ 1\ 1]^t$. (8)

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4. Attempt any **three** questions:

(a) Three approximate values of the number $\frac{1}{3}$ are given as 0.30, 0.33 and 0.34. Which of these three is the best approximation? Justify. (5)

(b) Determine the number of real and complex roots of the polynomial

$$f(x) = x^4 - 4x^3 + 3x^2 + 4x - 4 = 0$$

using descarte's rule of sign method. (5)

(c) Define condition number of a matrix A. Determine the condition number of the matrix $\begin{pmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{pmatrix}$ using the maximum absolute row sum norm. (5)

(d) Write necessary and sufficient condition of Jacobi's method to converge for solving the system $AX = b$. Find the condition on k so that the Jacobi method converges for solving $AX = b$, where $A = \begin{pmatrix} 1 & 0 & k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{pmatrix}$ (5)