QP Code: 26837

(3 Hours) [Total Marks:75

N.B. (1) Answers to the TWO sections should be written in the SAME Answer book.

(2) Figures to the right indicate full marks.

(3) Use of non – programmable calculators / log tables is allowed.

(4) Symbols have their usual meaning unless otherwise stated.

SECTION I

MATHEMATICAL METHODS

1. (a) Find the Fourier series of the function 13

f(x) = x(x+1) if $-\pi < x < \pi$.

(b) Using the method of Laplace transform, solve the differential equation

$y'' + 2y' + 2y = 0$ subject to initial conditions $y(0) = 2, y'(0) = -6$.

OR

2. (a) If F(w) and G(w) are the fourier transforms of f(t) and g(t) respectively, 13
show that the fourier transform of their product is $\frac{1}{\sqrt{2\pi}}$ times convolution of their fourier transforms.

(b) Find the eigen-values and eigenvectors of

$A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$

3 (a) State and prove Cauchy's theorem for an analytic function f(z) on a closed contour C. 12

(b) Check whether the following functions are analytic :-

(i) $f(z) = z$

(ii) $f(z) = e^{(x+iy)}$

OR

4. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$ 12

(b) Evaluate $\int_{-\infty}^{+\infty} \frac{\sin x \, dx}{x}$

Turn Over

5. (a) Using Frobenius method determine the solution of- 12

$x(x-1)y'' + (3x - 1)y' + y = 0.$

(b) Solve the differential equation,

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

assuming $u(x, t) = \phi(x)\eta(t)$, Where 'a' is constant subject to condition $u(x,t = 0) = \Phi(x)$.

OR

6. (a) Solve the differential equation $y'' + (1 + x^2)y = 0$ by power series method. 12

(b) Solve the differential equation

$y'' - 4y = 10$

SECTION II

CLASSICAL MECHANICS

7 (a) What are generalized coordinates? Using transformation equations show that the kinetic energy of the system can be written as the sum of three homogeneous functions of generalized velocities – 12

$T = T_0 + T_1 + T_2$

Where T_0 is independent of generalized velocities T_1 is linear in velocities and T_2 is quadratic in velocities.

(b) A particle is moving in a plane. Obtain an expression for the time derivative of the angular momentum. Use plane polar coordinates.

OR

8 (a) Use De'Alembert's principle to obtain the Lagrange's equations of motion. 12

(b) A hoop is rolling without slipping down an inclined plane. The inclined surface makes an angle θ with the horizontal. Find the number of degrees of freedom and the constraint equations.

Turn Over

9 (a) State and prove virial theorem. 13

(b) Show that the particle describes a circular orbit under the influence of an attractive central force directed towards the point on the circle, and then the force varies as the inverse fifth power of the distance.

OR

10 (a) Obtain the equations of motion of small oscillations of a system around the position of the stable equilibrium. 13

(b) Determine the eigen frequencies and the eigen coordinates of a system with two degrees of freedom whose Lagrangian is

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2k}(x^2 + y^2) + \alpha xy \ldots . \alpha > 0$$

11 (a) Derive Hamilton's equations of motion from variation principle. 13

(b) Using Legendre transformation obtain the canonical equations of the Hamiltonian.

OR

12 (a) What are generating functions? Discuss various generating functions 13

(b) Show that: $\frac{\partial}{\partial t}[\phi, \psi] = \left[\frac{\partial \phi}{\partial t}, \psi\right] + \left[\phi, \frac{\partial \psi}{\partial t}\right]$.
