B. N .BANDODKAR COLLEGE OF SCIENCE, THANE
F.Y.B.Sc. THEORY EXAMINATION MARCH 2014
SEMESTER II
USMT 201

DURATION:2 HOUR **MARKS :60**

N.B.: 1 All questions are compulsory.

Q.1 Attempt any THREE.

(1) For $u, v \in R^3$, Prove that $\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2)$. (5)

(2) Find parametric equation of the line passing through the point (1, 2, 3) in the direction (7, −1, 9). (5)

(3) Define Norm of a vector in R^3. Find the value of k for $\| kv \| = 5$ and $\|v\| = 2$. Find the value of $\| v \|$ for $\|kv\| = 9$ and $k = -2$. (5)

(4) Convert the Polar equation $r = 1 + \cos\theta$ into Cartesian equation. (5)

(5) Find spherical co-ordinate system whose cylindrical co-ordinates are $(1, \frac{\pi}{4}, 2)$. (5)

(6) Define distance from the point to the plane. Find the distance from the point P=(1, 2, 3) & the plane is $2x + 3y + 4z = 5$. (5)

Q.2 Attempt any THREE

(1) Evaluate the limit (5)

(i) $\lim_{(x,y)\to(1,-2)} \frac{x^2-y^2}{x+y}$ (ii) $\lim_{(x,y)\to(3,1)} \frac{x+y-4}{\sqrt{x+y}-2}$

(2) State Sandwitch Theorem. Use it to find (5)

$\lim_{(x,y)\to(0,0)} (x+y)\sin\frac{1}{x+y}$.

(3) By using Limit along the Path test , Show that $\lim_{(x,y)\to(0,0)} \frac{-xy}{x^2+y^2}$ does not exist at (0,0) (5)

(4) Define Bounded set in R^2. (5)

Show that the set $A = \{ (x, y) \in R^2 / -1 \le x \le 1, 0 \le y \le 2 \}$ is bounded set.

(5) Define Continuity of the function $f: R^2 \to R$ at (a, b).Check whether the function , (5)

$$f(x,y) = \frac{x^2-y^2}{x-y} \quad \text{for } x \neq y$$
$$= 2 \quad \text{for } x = y$$

is Continous or not at the point (1,1).

(P.T.O.)

(6) Find the Level curves of $f(x,y) = y - x^2$ for $c = 0, 1, -2, 3$. (5)

Q.3 **Attempt any THREE.**

(1) Find Partial Derivative of $f : R^2 \to R$, for $f(x, y) = x^2 - 2y$ at $(-1,1)$ using definition. (5)

(2) Define (i) Mixed Derivatives theorem. (5)

(ii) Differentiability of function of two variable.

(3) Using the Chain Rule, Find $\frac{dz}{dt}$ at $t = 1$ for $z = f(x, y) = x^2 + y^2$, where $x(t) = \cos t + \sin t$, $y(t) = \cos t - \sin t$. (5)

(4) Define Directional derivatives. Find Directional derivatives of $f(x,y) = -x^2 + y$ at $(2, 3)$ in the direction $(1, 1)$. (5)

(5) Find $\frac{dy}{dx}$, if $f(x, y) = y\,e^x + x \sin y - 2 = 0$ at $(0,2)$, by implicit differentiation. (5)

$6x$

(6) Find the Local extreme values of $f(x, y) = 3x^2 + y^2 - 3xy + 6y - 4y$. (5)

Q.4 **Attempt any THREE.**

(1) Convert the Cylindrical Co-ordinates $(2, \frac{3\pi}{2}, -1)$ and $(2, \frac{\pi}{6}, 0)$ to Cartesian Co-ordinates. (5)

(2) Find the equation of the plane passing through $A = (0, -2, 1)$, $B = (2, 0, 2)$ & $C = (1, 1, -1)$. (5)

(3) Show that the following using ε- $\overset{\delta}{\cancel{\partial}}$ definition. (5)

$$\lim_{(x,y)\to(1,2)} 2x + 3y = 8$$

(4) Using Algebra of limits, Evaluate $\lim_{(x,y)\to(2,-2)} \sqrt{(x^2 + y^2 + 1)}$ (5)

(5) Find the equation of Tangent plane for the function $f(x, y) = xy^2 + x^2y$, at $(-1, 2)$. (5)

(6) Find the Gradient vector of function $f(x, y) = y^2 - 4x + 1$, at $(1,1)$. Evaluate the directional derivative of f at $(1, 1)$ along the direction $(1,-1)$. (5)

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