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Duration: 2 Hours **[Total Marks: 60]**

N.B.: 1. All questions are compulsory.
2. Make suitable assumptions wherever necessary and state the assumptions made.
3. Answers to the same question must be written together.
4. Numbers to the right indicate marks.
5. Draw neat labelled diagrams wherever necessary.
6. Use of Non-programmable calculators is allowed.

Q. 1 **Answer *ANY FOUR* of the following:** **16 M**

a Define the following.
i. Universal statements ii. Existential universal statements iii. Subset
iv. Cartesian products v. Relation

b Use the De-Morgan's laws to find the negation of each of the following statements.
i. Jon is rich and happy.
ii. Carlos will bicycle or run tomorrow.
iii. Merry walks or takes the bus to class.
iv. Ibrahim is smart and hardworking.

c Prove that , $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

d Let A ={a,b,c,d,e} and B ={a,b,c,d,e,f,g,h} Find,
i. $A \cup B$
ii. $A \cap B$
iii. A-B
iv. B-A

e List the members of these sets .
i. {x/ x is the real number such that $x^2 = 1$}
ii. {x/ x is the positive interger less than 12}
iii. {x/ x is the square of an interger & x <100}
iv. {x/ x is an interger such that $x^2 = 2$ }

f Let A= {3,4,6,7}, B={6,10} and C={4,7}.
Answers the following questions. Give reasons for your answers.
i. Is $B \subseteq A$?
ii. Is $C \subseteq A$?
iii. Is $C \subseteq C$?
iv. Is C is the proper subset of A?

g Let A(n) be "n is factor of 12"
B(n) be "n is factor of 6"
C(n) be "n <7 and n≠5"
And suppose that domain of n is z^+, the set of positive integers. Use the $\Rightarrow$ and $\Leftrightarrow$ symbols to indicates true relationships amongs A(n),B(n) and C(n).

h Using proof by contraposition, Prove that if n is an integer and 3n+2 is odd, then n is odd.

Q. 2 **Answer *ANY FOUR* of the following:** **16 M**

a Solve the recurrence relation
$a_n = a_{n-1} + 6a_{n-2}$, for n≥2 , $a_0 = 3, a_1 = 6$

b By using the negation prove that $\sqrt{2}$ is irrational.

c Define G : $J_5 \times J_5 \rightarrow J_5 \times J_5$ as follows : For all (a,b) $\in J_5 \times J_5$,
G(a,b) =((2a+1) mod 5,(3b-2) mod 5)
Find : G(4,4), G(2,1), G(3,2), G(1,5)

d Define prime numbers and composite number . Express the definition using symbols. Write first six prime and composite numbers .

e State Euclidian algorithm. Find the gcd (330,156) using Euclidian algorithm

f Use the method of induction prove that the sum of first natural number is $\frac{n.(n+1)}{2}$

g Let f and g be the functions from the sets of integers to the sets of integers. Define by f(x) =2x+3 and g(x) =3x+2. What is the composition for f and g? And what is the composition for g and f?

h What are the values of the following products .

i. $\prod_{i=0}^{10} i$ ii. $\prod_{i=5}^{8} i$

iii. $\prod_{i=1}^{100}(-1)^i$ iv. $\prod_{i=1}^{10} 2$

Q. 3 **Answer *ANY FOUR* of the following:** **16 M**

a i. In how many ways could the letters in the word STATISTICS be rearranged?

ii. In how many ways can the word REARRANGE be rearranged?

b Explain the properties of an Relation.

c Let A={a,b,c,d}, Let
R={(a,a),(a,b),(a,c),(b,b),(b,d),(b,c),(c,c),(c,a),(c,d),(d,d),(d,c),(d,b)}
Prove that R is an equivalence relation

d i. How many ways of the letters of the word MATHEMATICS be arrange in a row if M and A remain together as a unit.

ii. How many ways can four of letters of the word MATHEMATICS be selected and written in a row.

iii. How many ways can the letters of the word MATHEMATICS be arrange in a row if the letters THE must remain together as a unit.

e A pair of fair dice is thrown. Find the probability p that the sum is 10 or greter if i. 5 appears on the first die ii. 5 appears on at least one dia.

f A single card is drawn from an ordinary deck of 52 cards. Find the probability p that

i. The card is king
ii. The card is face card.
iii. The card is heart
iv. The card is face card and heart

g Draw the directed graph for the following relations .

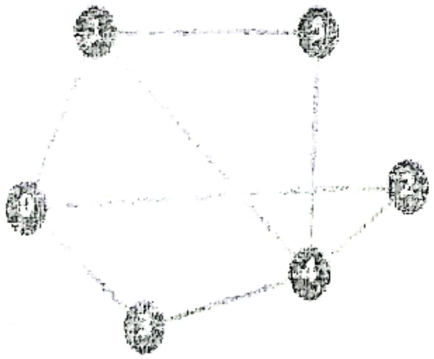
i. A relation R on A={1,2,3,4} BY R={(1,1),(2,3),(3,3)}
ii. Let A={2,3,4,5,6,7,8} and defined a relation R on A as follows.
For all x,y∈A, $_xR_y$ is x/y

h Define the following
I) Trail II) Path III) circuit IV) walk V) Tree

Q. 4 **Answer *ANY THREE* of the following:** **12 M**

a Find the value of i) 7! ii) P(6,3) iii) C(5,1)

b Find the adjacency matrix of the following graphs.



ii .Find the directed graph that have the following adjacency matrix.

1 0 1 2
0 0 1 0
0 2 1 1
0 1 1 0

c Suppose that f is defined recursively by
f(0) = 3, f(n+1)=2f(n)+3.
Find f(1),f(2),f(3),f(4) and f(5).

d For the following either draw the graph as per the specifications or explain why no such graph exists :

1. Grpah, circuit free,nine vertices,six edges.
2. Graph,two vertices,one edge ,not a tree.

e Define recurrence relation of the Fibonacci sequence also find the Fibonacci sequence f_2, f_3, f_4, f_5, f_6 where the initial condition $f_0 = 0, f_1 = 1$

f i) How many distinguishable ways can the letters of the word HULLABALOO be arranged in order ?
ii) How many distinguishable ordering of the letters of HULLABALOO begin with U and end with L ?

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