Time: 3 Hours Marks: 100

Note: 1) All questions are compulsory.
2) Use of non- programmable calculator is allowed.
3) Draw figures wherever necessary.
4) Symbols have their usual meanings unless mentioned.

Q.1 (A) Select the correct option 12

i) If $\vec{A} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} + 4\hat{k}$, then $|3\vec{A} + 2\vec{B}|$ is _____

(a) $\sqrt{150}$ (b) $\sqrt{15}$ (c) 150 (d) 25

ii) Curl of electrostatic field is _____
(a) non-zero (b) constant
(c) zero (d) variable function

iii) The order of a differential equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E \sin \omega t$ is _____.
(a) 0 (b) 1
(c) 2 (d) 3

iv) The time constant of a CR circuit with C=500μF and R=50KΩ is _____.
(a) 5 sec (b) 15 sec
(c) 25 sec (d) 35 sec

v) For a non-dispersive medium the relation between group velocity (v_g) and a phase velocity (v_p) _____.
(a) $v_g = v_p$ (b) $v_g = \lambda v_p$
(c) $v_g = kv_p$ (d) $v_g = Tv_p$

vi) _____ waves do not transmit energy from one point to another.
(a) Longitudinal (b) Transverse
(c) Progressive (d) Stationary

(B) Answer in one sentence : 03

i) What is the condition for the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ to be coplanar?

ii) Define Partial differential equation.

iii) Define Group velocity of a wave.

(C) Fill in the blanks 05

i) A vector whose initial and terminal points coincide is called _____ vector.

ii) The position vector of point P(2,2,2) with respect to origin is _____.

iii) The _____ in a series LR circuit is the time taken for the current to rise to 63% of its final maximum value.

iv) _____ can be defined as the periodic variation of intensity of wave at a given point due to superposition of two waves of slightly different frequencies travelling along the same direction.

926325B547BA5EC7DB1E74C07112B716

v) _____ is defined as oscillatory disturbance travelling through a medium.

Q.2 (A) Attempt any one 08

i) What do you understand by scalar field and vector field? Explain it with the help of suitable two examples of each.

ii) The linear velocity for the circular motion of the particle is $\vec{V} = \vec{\omega} \times \vec{r}$. Assuming ω to be constant, prove that, (a) $\nabla \times \vec{V} = 2\vec{\omega}$ and (b) $\nabla \times \vec{V} = 0$

(B) Attempt any one 08

i) What are scalar and vector triple product and give at least two properties of each.

ii) Explain the operator Del. Define the divergence of a vector field and explain its physical meaning.

(C) Attempt any one 04

i) Prove that $(\vec{A} \times \vec{B}).(\vec{C} \times \vec{D}) = (\vec{B}.\vec{D})(\vec{A}.\vec{C}) - (\vec{B}.\vec{C})(\vec{A}.\vec{D})$

ii) Suppose $\vec{A} = x^2z^2\hat{i} - 2y^2z^2\hat{j} + xy^2z\hat{k}$. Find curl $\vec{A}$

Q.3 (A) Attempt any one 08

i) Discuss the general first order linear differential equation with reference to complementary function and particular integral, Obtain its general solution.

ii) A series combination of a capacitor of capacitance (C) and a resistor of resistance (R) is connected across a source of constant emf (E). Derive an expression for the charge (q) on the capacitor and current through the circuit during charging of the capacitor.

(B) Attempt any one 08

i) Show that the following differential equation is exact and hence find its solution: $(2x + y + 1)\,dx + (2y + x + 1) = 0$.

ii) A charged capacitor discharges through a series combination of an inductor and a resistance. Derive an expression for the discharge of a capacitor. Under what condition is the discharge oscillatory?

(C) Attempt any one 04

i) A coil having a resistance of 15Ω and an inductance of 10H is connected to 90 volts supply. Determine the value of current after 2 seconds.

ii) If N is the number of radionuclei in a sample at time t, then the rate of radioactive decay is proportional to N itself. The differential equation describing the radioactive decay is $\frac{dN}{dt} = -\lambda N$, where λ is called the decay constant. Solve the equation.
(Given: N_0 is the initial number of radionuclei in the sample.)

926325B547BA5EC7DB1E74C07112B716

Q.4 (A) Attempt **any one** 08

i) Discuss the composition of two parallel simple harmonic motion of the same period and with same centre but of different amplitudes and different phase constants. Show that the resultant motion is also a simple harmonic motion having the same period. Find the expression for the amplitude and resultant motion.

ii) Define particle velocity and wave velocity.
Show that (particle velocity) = – (wave velocity) x (slope of the displacement curve).

(B) Attempt **any one** 08

i) Discuss the composition of two perpendicular simple harmonic motions of the same period and show that the path of the resultant motion, in general is an inclined ellipse.

ii) Derive the expression for the velocity of a simple harmonic wave in a stretched string.

(C) Attempt **any one** 04

i) The equation of motion of a particle is given by $x = 5\cos(3.14t + \frac{\pi}{6})$ in cm.
Find the period of oscillation and maximum acceleration of the particle.

ii) A guitar string is 90cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz.

Q.5 Attempt **any four** 20

i) Explain in brief (a) Collinear Vector (b) Position vector and Displacement vectors.

ii) Find the area of triangle having vertices A(1,3,2) ,B(2,-1,1) and C(-1,2,3).

iii) The given differential equation $(xy^2 + Ax^2y)\,dx + (x + y)\,x^2dy = 0$ is exact.
Find the value of A.

iv) In LR series circuit, L=100 mH and R=10Ω, is connected to a 50 volt battery. Calculate: (a) Time taken for the current to rise to half its steady voltage.
(b) Voltage drop across resistance after time t = 0.02 seconds.
(c) Voltage drop across inductance after time t = 0.02 seconds.

v) What are Lissajous figure? State the factors on which the shape of the Lissajous figures depends.

vi) A travelling wave propagates according to the given expression,
$y = 0.02 \sin(100x - 2t)$, where y is the displacement at position x and time t. Determine the amplitude, wavelength, frequency and period of the wave.

****

926325B547BA5EC7DB1E74C07112B716