B. N. BANDODKAR COLLEGE OF SCIENCE, THANE - 400 601.

FIRST TERM EXAMINATION OCT. - 2010

F. Y. B. Sc.

TIME : 2 Hrs.

SUBJECT : MATHEMATICS - II **MARKS :** 60

N. B. : 1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 a) Prove that ${}^{n}c_{r} + {}^{n}c_{r-1} = {}^{n+1}c_{r}$. [3]

b) Attempt ANY THREE of the following.

i) Prove by induction, n (n + 1) is divisible by 2. [4]

ii) Prove that the number of primes is infinite. [4]

iii) Show that $f: \mathfrak{R} \rightarrow \mathfrak{R}$, given by f (x) = 2x + 5 is a bijection and hence find its inverse. [4]

iv) Find the last digit of 7^{313}. [4]

v) Find the incongruent solutions of $6x \equiv 9 \pmod{21}$. [4]

Q.2. a) Prove by induction, $1 + 3 + 5 + \ldots\ldots + (2n - 1) = n^2$. [3]

b) Attempt ANY THREE of the following :

i) State and prove the Binomial Theorem for $n \in \mathbb{N}$, using the Principle of Induction. [4]

ii) Construct Pascal's triangle upto n = 5. [4]

iii) Prove that ${}^{n}c_{4} + 2\,{}^{n}c_{3} + {}^{n}c_{2} = {}^{n+2}c_{4}$. [4]

iv) State and prove Euclid's Lemma. [4]

v) Find the g.c.d of -1092 and 195. Also express it as a linear combination of - 1092 and 195. [4]

Q.3 a) Define injective, surjective and bijective map [3]

b) Attempt ANY THREE of the following.

i) Prove that composition of two injective functions is injective. [4]

ii) State properties of binary operations. [4]

iii) If A is a non empty subset of a finite set B then prove that $| A | < | B |$ [4]

iv) If 5 points are chosen in an equilateral triangle of side 2, show that at least two points are at a distance of atmost one unit from each other. [4]

v) State principle of addition and multiplication of counting. [4]

Q.4 a) Define congruence relation modulo n, with an example. [3]

b) Attempt ANY THREE of the following :

i) For any integer x, show that $x^2 \equiv 0$ or $1 \pmod 4$. [4]

ii) Define Euler ϕ - function. Find $\phi(360)$. [4]

iii) Verify Wilson's Theorem for $p = 7$. [4]

iv) Show that $2^{81} \equiv 2 \pmod{41}$.. [4]

v) Prove that $\phi(mn) = \phi(m)\,\phi(n)$ if $(m, n) = 1$. [4]

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