# VPM 's B. N. Bandodkar College of Science, Thane

## Fourth Semester End Theory Examination March 2015

## S.Y.B.Sc.Mathematics Paper II( USMT 402)

Duration: 2 1/2 hours Max. Marks: 75

N.B. 1) All questions are compulsory

2) Figures to the right indicate full marks.

Q.1 (a) Define rank and nullity of a linear transformation. State and prove rank-nullity theorem. (8)

or

(a) If A is a linear map of V to itself such that $A^2 - A + I = 0$. Show that A is invertible, and $A^{-1} = I - A$. (8)

(b) **Attempt any three of the following questions**

(i) Show that the map T: $IR^3 \rightarrow IR^3$ ; T(x, y, z) = (x , y, 0) is linear. (4)

(ii) If F: $IR^3 \rightarrow IR^3$ be linear transformation defined by F(x, y, z ) = (x ,x+y,0) Find the matrix of F w. r. t. basis B = {(1,1,0),(0,1,1),(1,1,1)} on both sides. (4)

(iii) If A, B be linear maps of V into itself. If Ker A = Ker B = {0}, show that Ker AoB = {0}. (4)

(iv) If F: $IR^3 \rightarrow IR^3$ be defined by F(x, y, z ) = (x + y + z, x + 2y - z, 3x + 5y - z). Find if F is non singular. (4)

Q.2 (a) Prove that there exists a multilinear alternating function f: $IR^n$ x ...,$IR^n$ $\rightarrow$IR such that f(I) = 0. (8)

or

(a) For $A^1, A^2,\ldots,A^n$ n linearly dependent column vectors in $IR^n$ , prove that Det $(A^1, A^2,\ldots,A^n)$ = 0. (8)

(b) **Attempt any three of the following questions**

(i) Show that similar matrices have same determinant. (4)

(ii) Give geometric meaning of 2 x 2 and 3 x 3 determinant. Prove that the vectors (3,0) and (4/7, 0) are collinear. (4)

(iii) Find determinant of $A = \begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$ using axiomatic definition. (4)

(iv) Using permutation , prove that Det (A) = Det ($A^T$), for a square matrix A. (4)

Q.3 (a) Find eigenspace of $A = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$. (8)

or

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(a) Find eigenvectors of $A = \begin{bmatrix} 2 & 3 \\ 4 & -2 \end{bmatrix}$. (8)

**(b) Attempt any three of the following questions**

(i) Define characteristic polynomial, eigenvalue and eigenvector for a matrix A. Show that if $\lambda$ is eigenvalue of A, then $\lambda^{-1}$ is eigenvalue of $A^{-1}$. (4)

(ii) Find characteristic polynomial for $A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$. (4)

(iii) For an n x n matrix A with eigenvalue $\lambda$, prove that $E_\lambda$ is a subspace of $IR^n$. (4)

(iv) Prove that for any n x n square matrix A, A and $A^T$ have same eigenvalues. (4)

Q.4 **Attempt any three of the following questions**

(i) For a n x n square A, if $\lambda$ is an eigenvalue of A, then prove that $\lambda^2$ is eigenvalue of $A^2$. (5)

(ii) If $L: V \rightarrow V$ be a linear map such that $L^2 + 2L + I = 0$. Show that L is invertible. (5)

(iii) For n x n matrix A, prove that $\lambda$ is eigenvalue of A if and only if $\lambda$ is a root of the characteristic polynomial of A. (5)

(iv) Define KerT and ImT for a linear transformation T. Find Ker T for (5)

$T: IR^4 \rightarrow IR^3$ ; $T(x, y, z, u) = (x - y + z + u, x + 2z - u, x + y + 3z - 3u)$.

(v) Prove using permutations, (5)

$Det(A^1, A^2, \ldots, A^i, \ldots, A^j, \ldots, A^n) = - Det(A^1, A^2, \ldots, A^j, \ldots, A^i, \ldots, A^n)$.

(vi) Solve the system of equations $2x - y + z = 1$, $x + 3y - 2z = 0$, $4x - 3y + z = 2$, using Cramer's rule. (5)

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