B. N. BANDODKAR COLLEGE OF SCIENCE – THANE
S.Y.B.Sc. Theory Examination March 2015
Semester IV
USMT 401

DURATION:2 $\frac{1}{2}$ HOURS

MARKS :75

N.B.: 1 All questions are compulsory.
2 figures on right side indicates max. marks.

Q.1 A) Attempt any One. (8)

(i) a) Solve the differential equation $(e^y + 1)\cos x\ dx + e^y \sin x\ dy = 0$ (4)

b) Solve the differential equation $y'' + y = 2e^{2x} + 3x^2$ by undetermined coefficient. (4)

(ii) p) Solve the differential equation $(D^2 - 7D + 6)y = 2\sin 3x$ (4)

q) Solve the differential equation (4)

$$(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 + 5y^4)dy = 0$$

B) Attempt any Three. (12)

i) Solve the differential equation $(x + y + 4)\ dx = (x + y - 6)dy$ (4)

ii) Solve the differential equation $(xy)dx + (1 + x^2)dy = 0.$ (4)

iii) Solve the differential equation $y'' - 3y' + 2y = e^{3x}$ (4)

iv) If a sample of some radioactive element is reduced from 1 gram to 0.91 gram after 37 days. What is half life of the element? (4)

Q.2 A) Attempt any One. (8)

(i) a) Find the volume of the solid lies under the elliptic paraboloid $\frac{x^2}{4} + \frac{y^2}{9} + z = 1$ and above the rectangle R= $[-1,1]\times[-2,2]$. (4)

b) Find the area enclosed by Cardioid $r = a(1 + \cos\theta)$ (4)

(ii) p) Find area of the region bounded by $y = 2x^2$ & $y^2 = 4x$. (4)

q) Find the volume of tetrahedron enclosed by the co-ordinate planes & the plane $2x + y + z = 4$ (4)

B) Attempt any Three. (12)

i) Evaluate $\iint x^2 + y^2\, dA$, over the region bounded by $x^2 = y$ & $y^2 = x$. (4)

ii) Use polar co-ordinates to find a double integral $\int_0^2 \int_0^{\sqrt{4-x^2}} x^2 + y^2\ dy\ dx.$ (4)

iii) Evaluate $\int_0^{2\pi} \int_0^{\pi} \int_0^{2\sin\varphi} \rho^2 \sin^2\varphi\ d\rho\ d\varphi\ d\theta.$ (4)

iv) Find mass of a unit cube of density $\rho(x,y,z) = x + 2y + 3z$. (4)

P.T.O.

Q.3 A) **Attempt any One.** **(8)**

(i) a) Evaluate $\int_c ydx - xdy$ over the rectangle [-1, 1] X [-1, 1]. (4)

b) Find the flow of velocity field $F = (x+y)\vec{\imath} - (x^2+y^2)\vec{\jmath}$ along the path from (1,0) to (−1,0) in xy plane along the line segment from (1,0) to (−1,0). (4)

(ii) p) Let $F = (x^2+y^2)\vec{\imath} + (3xy - y^2)\vec{\jmath}$, Find the $curl\ F(x,y)$ & $div\ F$. (4)

q) Find the work done by the force $F = 3y\ \vec{\imath} + 2x\ \vec{\jmath} + \ 4z\ \vec{k}$ along the curve $r(t) = t\ \vec{\imath} + t\ \vec{\jmath} + \ t\,\vec{k}, 0 \le t \le 1$, from (0,0,0) to (1,1,1). (4)

B) **Attempt any Three.** **(12)**

i) Find the mass of the wire that lies along the curve $r(t) = \sqrt{2}t\ \vec{\imath} + \sqrt{2}t\ \vec{\jmath} + (4 - t^2)\,\vec{k}, 0 \le t \le 1$, if the density $\delta = 3\,t$. (4)

ii) Determine whether a vector field $F = (z+y)\vec{\imath} + (z)\vec{\jmath} - (x+y)\vec{k}$ is conservative or not. (4)

iii) Evaluate $\int_c (2 + x^2y)ds$, where C is the upper half of the unit circle $x^2 + y^2 = 1$. (4)

iv) Find the flux of the field $F = x\,\vec{\imath} + y\vec{\jmath}$ outward the ellipse having parameterizations $r(t) = \cos t\,\vec{\imath} + 4\sin t\,\vec{\jmath}\ , 0 \le t \le 2\pi$. (4)

Q.4 **Attempt any Three.** **(15)**

i) Solve the differential equation $y'' - 2y' + y = x^{3/2}e^x$ by variation parameter. (5)

ii) Solve the differential equation $(xy^2)dx - (x^3 + y^3)dy = 0$ (5)

iii) Evaluate $\iiint_E \sqrt{x^2+y^2}\,dV$ in cylindrical co-ordinates where E is solid lies between the cylinder $x^2 + y^2 = 1$ & $x^2 + y^2 = 4$ above x-y plane & below the plane z = x + 2. (5)

iv) Find the volume of a solid which is bounded by the curves $y^2 = x, x^2 = y$ & the planes z = 0, x + y + z = 2. (5)

v) Use green's theorem to evaluate $\int_c xdx + xydy$, where C is the circle with radius 1. (5)

vi) Evaluate $\int_c (xy + y + z)\ ds$, along the curve C parameterized by $r(t) = 2t\,\vec{\imath} + t\,\vec{\jmath} + \ (2 - 2t)\,\vec{k}, 0 \le t \le 1$. (5)

****