V.P.M's B. N. Bandodkar College of Science, Thane

S.Y.J.C Preliminary Examination January 2020

Subject: Mathematics and Statistics

Date: 17.01.2020 **Time: 02.30 pm to 05.30 pm**

Day: Friday **Marks: 80**

Note: The question paper consists of 34 questions divided into four sections, namely A, B, C and D.

1) **Section A: Q. No.1 contains 8 multiple choice type of questions carrying TWO marks of each. Q. No.2 contains 4 very short answer type of questions carrying ONE mark each.**
2) **Section B: Q. No.3 to Q. No.14 contains 12 short answer type of questions carrying TWO marks each out by which Eight are to be attempted.**
3) **Section C: Q. No.15 to Q. No.26 contains 12 short answer type of questions carrying THREE marks each out by which five are to be attempted.**
4) **Section D: Q. No.27 to Q. No.34 contains 8 long answer type of questions carrying FOUR marks each out of which any five are to be attempted .**
5) **Use of log table is allowed. Use of calculator is not allowed.**
6) **Figures to the right indicate full marks.**
7) **Use of graph paper is not necessary. Only rough sketch is expected.**
8) **Star each section on new page.**
9) **For each MCQ, the correct answer must be written along with its alphabet. eg. (a).../(b).../(c).../(d).. etc.**
10) **Evaluation of each MCQ would be done for the first attempt only.**

Section - A

Q.1 Select and write the most appropriate answer from the given alternatives in each question: 16M

i) If p: girls are happy, q : girls are playing, then the symbolic from of the statement "Either the girls are happy or they are not playing" is.....

a) $p \vee \sim q$ b) $p \wedge \sim q$ c) $p \rightarrow \sim q$ d) $p \leftrightarrow \sim q$

ii) The probability distribution function of a random variable X, is given by

$f(x) = \frac{x^2}{18}$, for $-3 < x < 3$

$= 0$, otherwise

Then the value of $p(X < 1)$ is

a) $\frac{1}{9}$ b) $\frac{14}{27}$ c) $\frac{8}{9}$ d) $\frac{1}{6}$

iii) Direction cosines of the line pasting through the points A(-4,2,3) and B(1,3,-2) are

a) $\pm\frac{1}{\sqrt{51}}, \pm\frac{5}{\sqrt{51}}, \pm\frac{1}{\sqrt{51}}$ b) $\pm\frac{5}{\sqrt{51}}, \pm\frac{1}{\sqrt{51}}, \pm\frac{-5}{\sqrt{51}}$

c) $\pm 5, \pm 1, \pm\sqrt{5}$ d) $\pm\sqrt{51}, \pm\sqrt{51}, \pm\sqrt{51}$

iv) The equation of line passing through the point with position vector $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and parallel to the line joining the points with position vectors $-\hat{\imath} + 4\hat{\jmath} + \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ is

a) $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$ b) $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{-1}$ c) $\frac{x-2}{-2} = \frac{y+1}{-2} = \frac{z-1}{1}$ d) $\frac{x+2}{2} = \frac{y-1}{-2} = \frac{z+1}{1}$

v) The principle solution of the equation $cotx = -\sqrt{3}$ is _____.

a) $\frac{\pi}{3}, \frac{\pi}{6}$ b) $\frac{5\pi}{6}, \frac{3\pi}{2}$ c) $\frac{2\pi}{3}, \frac{5\pi}{3}$ d) $\frac{5\pi}{6}, \frac{11\pi}{6}$

vi) The value of $\int cos^2x.sin^2xdx$ is _____.

a) $\frac{1}{8}\left(x - \frac{sin4x}{4}\right) + C$ b) $\frac{1}{4}\left(x - \frac{sin4x}{4}\right) + C$

c) $sec^2x + C$ d) $tanx + C$

vii) The point of intersection of the lines represented by $x^2 - 9y^2 - x + 3y = 0$ is ____.

a) $\left(\frac{1}{2}, \frac{1}{6}\right)$ b) $\left(\frac{1}{2}, \frac{-1}{6}\right)$ c) $\left(\frac{-1}{2}, \frac{1}{6}\right)$ d) $\left(\frac{-1}{2}, \frac{-1}{6}\right)$

viii) If $y = sec^{-1}\left(\frac{\sqrt{x}-1}{x+\sqrt{x}}\right) + sin^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right)$ then $\frac{dy}{dx}$ is _____.

a) x b) $\frac{1}{x}$ c) 1 d) 0

Q.2 Answer the following: **04M**

i) In a binomial distribution, the probability of getting success is $\frac{1}{4}$ and standard deviation is 3, then find mean.

ii) Find the vector equation of the plane passing through a point having position vector $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and perpendicular to the vector $4\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$.

iii) If $y = x^{e^x}$ then find $\frac{dy}{dx}$

iv) Find the value of 'λ' for which the points (6, -1, 2), (8, -7, λ) and (5, 2, 4) are collinear.

Section - B

Attempt any Eight of the following: **16M**

Q.3 In ΔABC, prove that $a(bcosC - acosB) = b^2 - c^2$.

Q.4 Using truth table, prove the following logical equivalence $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

Q.5 Differentiate $tan^{-1}\left(\frac{1-sinx}{cosx}\right)$ w.r.t. x.

Q.6 Determine K such that the following function is a p.m.f

$$p(x) = \begin{cases} K\binom{4}{x}, & x = 0,1,2,3,4 \quad k > 0 \\ 0 & otherwise \end{cases}$$

Q.7 A stone is dropped into a pond waves in the form of circles are generated and the radius of the outermost ripple increases at the rate of 2 inches / sec. How fast is the area increasing when **(i)** the radius is 5 inches? **(ii)** after 5 seconds

Q.8 Solve the D.E $\left(y + x\frac{dy}{dx}\right)\sin xy = cosx$ using the substitution $xy = u$. Also find the particular solution if $y = 0$, when $x = 0$.

Q.9 Evaluate $\int \frac{1}{\sqrt{2x^2+3x+5}}dx$

Q.10 If $\hat{\imath} + \hat{\jmath} + \hat{k}, \hat{\imath} - \hat{\jmath} + \hat{k}$ and $2\hat{\imath} + 3\hat{\jmath} + m\hat{k}$ are coplanar then find m.

Q.11 Find the vector equation of the line passing through the point (-1, -1, 2) and parallel to the line $2x - 2 = 3x + y = 6z - 2$

Q.12 Find the value of K, if the following equation represent a pair of lines $3x^2 + 10xy + 3y^2 + 16y + k = 0$

Q.13 A fair coin is tossed 9 times. Find the probability that it shows heads.
(i) Exactly 5 - times **(ii)** in the first four tosses & tails in the last five tosses.

Q.14 Find the value of K, so that the function $f(x)$ continuous at the indicated point

$$\left.\begin{array}{l} f(x) = \frac{Log(1+kx)}{sinx}, for\ x \neq 0 \\ = \quad 5 \quad , for\ x = 0 \end{array}\right\} at\ x = 0$$

Section - C

Attempt any Eight of the following: **24M**

Q.15 Construct switching circuit for $[p \wedge \sim q] \vee [\sim p \wedge q] \vee [\sim p \wedge \sim q]$ also draw simplified switching circuit.

Q.16 Find the general solution of $sin2x + sin4x + sin6x = 0$

Q.17 Show that the volume of the parallelopiped whose co-terminus edges are $\bar{a}\ \bar{b}\ \bar{c}$ is $[\bar{a}\ \bar{b}\ \bar{c}]$

Q.18 If the direction ratios of two vectors are connected by the relation $p + q + r = 0$ and $p^2 + q^2 + r^2 = 0$. Find the angle between them.

Q.19 Find the vector equation of the line passing through the point whose position vector is $3\hat{\imath} + \hat{\jmath} - \hat{k}$ and which is parallel to the vector $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$. If P is a point on this line such that AP = 15. Find the position of vector of P.

Q.20 Find the vector equation of the plane passing through the points $\hat{\imath} + \hat{\jmath} - 2\hat{k}$, $\hat{\imath} + 2\hat{\jmath} + \hat{k}$, $2\hat{\imath} - \hat{\jmath} + \hat{k}$.

Q.21 If y is a differentiable function of 'u' and u is differentiable function of x, then prove that y is a differentiate function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Q.22 Evaluate : $\int \frac{1}{2-3sin2x} dx$

Q.23 If $f(-x) = -f(x)$ then prove that :

$$\int_{-a}^{a} f(x)dx = 0$$

Q.24 Find the area of the region bounded by the parabola $y = x^2$ and the line $y = x$ in the first quadrant.

Q.25 The Probability of hitting a target in any shot is 0.2. If 10 shots are fired, find probability that the target will be hit at least twice.

Q.26 Find K, if the function f defined by

$f(x) = kx(1-x)$ $\quad 0 < x < 1$

$= 0$ $\quad otherwise$

is the p.d.f of a.r.v 'X'.

Also find $p\left(X < \frac{1}{2}\right), p\left(\frac{1}{4} < X < \frac{1}{2}\right)$

Section - D

Attempt any FIVE of the following: **20M**

Q.27 Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ by using elementory row transformation.

Q.28 If $f(x)$ is continuous on $[0, \pi]$ where

$$f(x) = \begin{cases} x + a\sqrt{2}sinx & , for\ 0 \le x \le \frac{\pi}{4} \\ 2xcotx + b & , for\ \frac{\pi}{4} < x < \frac{\pi}{2} \\ acos2x - bsinx & , for\ \frac{\pi}{2} \le x \le \pi \end{cases}$$

Find the value of 'a' and 'b'.

Q.29 Minimize $Z = 6x + 4y$, subject to $3x + 2y \ge 12, x + y \ge 5, 0 \le x \le 4, 0 \le y \le 4$.

Q.30 If the population of a country doubles in 60 years, in how many years will it be triple under the assumption that the rate of increases is proportional to the number of inhabitants? (Given log2 = 0.6912, log3 = 1.0986)

Q.31 If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax + 5, x \in [1,3]$ with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

Q.32 Evaluate : $\int \frac{x^2}{(x^2+2)(2x^2+1)} dx$

Q.33 In ΔABC, if $\angle C = \frac{\pi}{2}$, then prove that $\sin(A-B) = \frac{a^2-b^2}{a^2+b^2}$

Q.34 Determine the maximum and minimum values for the function.

$f(x) = 2x^3 - 21x^2 + 36x - 20$

~*~*~*~*~*~*~*~*~*~*~*~*~