T.Y. B.Sc Paper II (Unit I and II) question Bank

Unit I

1. Explain the problem of Estimation.

2. Define minimum variance unbiased estimator (MVUE). Prove that if it exists it is unique.

3. Distinguish between parameter and statistic used in theory of estimation.

4. Explain the terms i) Point estimation ii) Interval estimation iii) Relative efficiency

5. Distinguish between i) estimator and estimate ii) Point estimation and interval estimation iii) Consistency and Efficiency

6. If X_1 ---------- Xn is a random sample fro0m B (1, p) show that i) T(T-1)/{n(n-1)} is an unbiased estimator of p^2 where T=∑ Xi ii) T(n-T)/{n(n-1)} is an unbiased estimator of p(1-p).

7. Based on a random sample of size n, obtain a sufficient estimator of θ for the following p.d.f.

$$f(x, \theta) = \frac{e^{-x/\theta}}{\theta} \qquad x>0, \quad \theta>0$$

$$= 0 \qquad \text{o.w.}$$

8. Define sufficient statistic. State Neyman's factorization theorem.

9. State and prove the sufficient condition for consistency

10. If X1 ------ Xn is random sample from Poisson distribution with parameter θ Use Neyman's factorization theorem to obtain sufficient estimator of θ.

11. Explain with the help of an example the use of Neyman's factorization theorem in obtaining sufficient estimator of a parameter .

12. Explain the following terms with suitable example. i) Unbiasedness ii) Consistency

    iii) Efficiency.

13. Suppose X1 ------Xn is a random sample from N (θ, 1) population. Give example of an estimator of θ which is i) both consistent and unbiased ii) Consistent but not unbiased.

14. Obtain sufficient estimator of θ based on a random sample of size n if

i) $f(x, \theta) = 1/\theta \qquad 0 < x < \theta$

$\quad = 0 \qquad$ o.w

ii) $f(x, \theta) = e^{-(x-\theta)} \qquad x > \theta$

$\quad = 0 \qquad$ o.w.

iii) $f(x, \theta) = \theta X^{\theta-1} \qquad 0 < x < 1, \ \theta > 0$

$\quad = 0 \qquad$ o.w.

iv) $f(x, \theta) = (1/\theta)\ e^{-(x/\theta)} \qquad X > 0, \ \theta > 0$

$\quad = 0 \qquad$ o.w.

15. Comment on the following statement giving suitable example.

    i) An unbiased estimator is unique.

    ii) A consistent estimator is unique.

16. Let X_1 -----X_n denote a random sample from the distribution

    $f(x, \theta) = 1/\theta \qquad 0 < x < \theta, \ \theta > 0$

    $\quad = 0 \qquad$ o.w.

    Two estimation suggested to estimate of θ are $T_1 = 2\overline{X}$ and $T_2 = (n + 1) X(n)/n$, $X(n) = \text{Max}(X_1 \text{---} X_n)$ of these two estimation which is better ? & Why ?

17. Let X_1 ---- Xn denote a random sample from normal population with mean zero and variance θ. Check whether sample variance is consistent for θ.

18. If t1 and t2 are consistent estimator of $g_1(\theta)$ and $g_2(\theta)$ respectively , show that at_1+bt_2 is consistent estimator of $ag_1(\theta) + bg_2(\theta)$ where a and b are constant.

19. If a random variable X follows Poisson distribution with mean λ . Show that $(-3)^x$ is a unbiased estimator of $e^{-4\lambda}$. Comment.

20. If T is an unbiased estimator of θ then show that $\sqrt{T}$ and T^2 are biased estimation of $\sqrt{\theta}$ and θ^2 respectively

21. If T is a consistent estimator of θ . Then show that $\sqrt{T}$ and T^2 are consistent estimators of $\sqrt{\theta}$ and θ^2 respectively .

22. X1 -----Xn be a random sample from Poisson distribution with parameter m. Obtain an unbiased estimator for i) m ii) m^2.

23. State and prove Cramer-Rao inequality . Also state the conditions under which it holds.

24. Let X1 ----Xn be a random sample selected from a population

$$f(X,.\Theta) = \Theta\ (1 - \theta)^{x-1} \qquad x = 1, 2\text{------}\ 0 < \Theta < 1$$

$$= 0 \qquad \text{o.w.}$$

show that sample mean attains CRLB.

25. Test whether minimum variance unbiased estimator for Θ exists

$$f(x\ \theta) = \frac{1}{\pi(1+(x-\theta)^2)} \qquad -\omega < x < \omega\ , \quad \omega < \theta < \omega$$

26. Let X_1 --- Xn be a random sample from a population with p.d.f.

$$f(x,\theta) = \frac{2xe^{-x^2/\theta}}{\theta} \qquad x > 0 \quad \Theta > 0$$

$$= 0 \qquad \text{o.w.}$$

Obtain CRLB for variance of an unbiased estimator of Θ. Examine whether it is attained .

27. Let X_1 --- Xn is a random sample from N (0., σ^2) . Examine whether $T = \sum xi^2$ is MVUE of σ^2.

28. Let X_1 --- Xn be a random simple from Binomial with parameters (K. p) . (K. known). Obtain MVUE of p . Also obtain its variance

29. Let X_1 --- Xn be a random simple from exponential population with parameterθ .Find Cramer Rao Lower bound for the variance of an unbiased estimator of $1/\theta$ .Also obtain MVUE of $1/\theta$

30. Let X_1 --- Xn be a random sample from a population

$$f(x, \theta) = \frac{\theta^{a} x^{a-1} e^{-\theta x}}{\Gamma a} \qquad X > 0, \; \Theta > 0$$

$$= 0 \qquad \text{o.w.}$$

Find CRLB for the variance of an unbiased estimator of 1/ Θ. Examine whether it is attained.

Unit II

31. Let probability density function (p.d.f.) of a random variable (r.v.) X be f(x, θ), and a random sample of n observations from a population with this p.d.f. is given. Obtain the estimator of the parameter θ by a) method of moments b) method of maximum likelihood and estimate standard error of the m.l.e,

i) $f(x, \theta) = \theta e^{-\theta x}$; $x > 0, \theta > 0$
$= 0$; otherwise.

ii) $f(x, \theta) = \theta x^{\theta - 1}$; $0 < x < 1, \theta > 0$.
$= 0$; otherwise.

iii) $f(x, \theta) = \frac{x}{\theta^2} e^{-x/\theta}$; $x > 0, \theta > 0$.
$=0$ otherwise

iv) $f(x, \theta) = 2\theta x e^{-\theta x^2}$; $x > 0, \theta > 0$
$= 0$; otherwise

v) $f(x, \theta) = (1+\theta) x^{\theta}$; $0 < x < 1, \theta > 0$.
$= 0$; otherwise.

32. Given the p.d.f. f(x, θ) of a r.v. X, and a random sample of n observations from a population with this p.d.f., estimate the unknown parameter θ by (a) method of moments (b) method of maximum likelihood

i) $f(x, \theta) = \frac{1}{\theta}$; $0 < x < \theta$
$= 0$; otherwise.

ii) $f(x, \theta) = e^{-(x-\theta)}$; $\theta < x < \infty , -\infty < \theta < \infty$
$= 0$; otherwise.

iii) $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}$; $-\infty < x < \infty , -\infty < \theta < \infty$
$= 0$; otherwise.

33. Let a random sample of n observations be taken from a population with p.d.f.

$$f(x, a, b) = \frac{1}{\beta - \alpha} \quad ; \ \alpha < x < \beta \ , \ -\infty < \alpha, \beta < \infty$$

Estimate the parameters α and β by a) method of moments and b) method of maximum likelihood.

34. Let sample of n observations be taken from a population with p.d.f

$$f(x, a, b) = \frac{1}{b} e^{-(x-a)/b} \quad ; \ a < x < \infty \ , \ -\infty < a < \infty \ , \ b > 0$$

Estimate the parameters a and b by a) method of moments and b) method of maximum likelihood.

35. Let sample of n observations be taken from a population with p.d.f

$f(x, \theta) = 1$ $\theta < X < \theta+1$

$=0$ o.w

Obtain m.l.e of θ

36. A sample of size n was taken from a normally distributed population having mean μ and variance σ^2, i.e. $X \sim N(\mu, \sigma^2)$. Find

a) Maximum likelihood estimate (m.l.e) of (i) μ when σ^2 is given. (ii) σ^2 when μ is given.
b) m.l.e's of μ and σ^2.

37. Data is drawn from a Negative binomial distribution with parameters k and p obtain the estimators of the parameters k and p by method of moments.

38. A random sample is drawn from a geometric distribution with parameter p obtain the estimator of the parameter p by method of moments.

39. A random sample is drawn from a Poisson distribution with parameter λ obtain the estimator of the parameter by i) method of moments ii) method of maximum likelihood.

40. A random sample is drawn from a Bernoulli distribution with parameter p obtain the estimator of the parameter by i) method of moments ii) method of maximum likelihood.

41. Let $X_1, X_2 \ldots\ldots\ldots X_n$ be a random sample from beta distribution with parameters (a, b). obtain the estimator of the parameters by method of moments.

42. Let $X_1, X_2 \ldots\ldots\ldots X_n$ be a random sample from gamma distribution with parameters (a, b). obtain the estimator of the parameters by method of moments.

43. State the important properties of m.l.e.

44. Prove that if sufficient statistic exists it is a function of m.l.e.

45. Explain the following methods of Point estimation. i) method of moments ii) method of maximum likelihood iii) method of modified chi-square.

46. Comment on the following statement giving suitable example.

    MLE is unique

47. The distribution of lifetimes of bulbs made by a certain manufacturer is exponential with mean $1/\theta$. Suppose θ has a priori exponential distribution with mean 1. Using squared error loss function, find Bayes' estimate of θ.

48. The time (in minutes) required to serve a customer at a certain facility has an exponential distribution, for which the value of the parameter θ is unknown. The prior distribution of θ is a gamma distribution with parameters (5, 2). Using squared error loss function, find Bayes' estimate of θ.

49. The brain weight (in grams) of adults is assumed to have a normal distribution $N(\mu, \sigma_0^2)$, a sample of size n is selected from this population. The experimenter believes that μ itself has a normal distribution $N(\theta, 1)$. θ known Find Bayes' estimate of μ using squared error loss function. What is the Bayes' estimate of μ if absolute error loss function is used?

50. Let $X_1, X_2 \ldots\ldots\ldots X_n$ be a random sample from Poisson distribution with parameter θ. The prior distribution of θ is an exponential distribution with mean 1. Using squared error loss function, find Bayes' estimate of θ.

51. Let $X_1, X_2 \ldots\ldots\ldots X_n$ be a random sample from binomial distribution with parameters $(1, \theta)$. The prior distribution of θ is beta distribution with parameters (a, b). Using squared error loss function, find Bayes' estimate of θ.

52. Let $X_1, X_2 \ldots\ldots\ldots X_n$ be a random sample from Poisson distribution with parameter θ. The prior distribution of θ is gamma distribution with parameters(α, β). Using squared error loss

function, find Bayes' estimate of θ.Show that it is a weighted average of m.l.e. and the prior mean.

53. Let $X_1, X_2 \ldots\ldots\ldots X_n$ be a random sample from normal distribution with parameters $N(\mu,1)$. Parameter μ itself has a normal distribution $N(0, 1)$. i) Find Bayes' estimate of μ using squared error loss function. Also show that risk=1/(n+1). ii)What is the Bayes' estimate of μ if absolute error loss function is used? Show that risk= $\sqrt{\frac{2}{\pi(n+1)}}$

54. Let $X_1, X_2 \ldots\ldots\ldots X_n$ be a random sample from binomial distribution (1,p) .Prior distribution of p is given by

$h(p)= \alpha p^{\alpha-1} \quad 0 < p < 1$

obtain Bayes' estimate of p using squared error loss function.

55. Let $X_1, X_2 \ldots\ldots X_n$ be a random sample from normal distribution with parameters $N(\mu,\sigma_0^2)$. Parameter μ itself has a normal distribution $N(\theta, 1)$. i) Find Bayes' estimate of μ using squared error loss function. State prior mean and m.l.e.of μ. Also show that as sample size increases, the importance of prior mean decreases and importance of m.l.e. increases.

56. Let $X_1, X_2 \ldots\ldots X_n$ be a random sample from normal distribution with parameters $N(\mu,\sigma^2)$, σ^2 known. μ itself has a normal distribution $N(\theta, \sigma_0^2)$. θ known Find Bayes' estimate of μ using squared error loss function. What is the Bayes' estimate of μ if absolute error loss function is used?

57. Let $X_1, X_2 \ldots\ldots X_n$ be a random sample from normal distribution with parameters $N(\mu,1)$, μ itself has a normal distribution $N(\theta, \sigma_0^2)$, θ known. Find Bayes' estimate of μ using squared error loss function. What is the Bayes' estimate of μ if absolute error loss function is used?

58. Define i) Loss function i) Risk function iii) Prior distribution iv) Posterior distribution.

59. State the asymptotic property of m.l.e. Explain how this property is used to obtain an interval estimate of the parameter.

60. Let $X_1, X_2 \ldots\ldots\ldots X_n$ (n<30) be a random sample from normal distribution with parameters $N(\mu, \sigma^2)$, construct $100(1-\alpha)$ % confidence interval for

    i) μ when σ^2 known

    ii) μ when σ^2 unknown

    iii) σ^2 when μ known

iv) σ^2 when μ unknown

61. Describe a method of obtaining a confidence interval for a parameter based on the large sample. Obtain 100(1-α) % confidence interval for θ , the mean of an exponential distribution.

62. Let $X_1, X_2 \ldots\ldots\ldots X_n$ (n>30) be a random sample from normal distribution with parameters $N(\mu, \sigma^2)$, construct 100(1-α) % confidence interval for

i) μ when σ^2 known ii) μ when σ^2 unknown

63. Based on two independent random samples selected from two Normal populations . construct 100(1-α) % confidence interval for $(\sigma_1^2 / \sigma_2^2)$

i) when population means are known

ii) population means are unknown

64. Obtain 100(1-α) % confidence interval for θ , the mean of a Poisson distribution using asymptotic property of maximum likelihood estimators.

65. Obtain 100(1-α) % confidence interval for θ , the parameter of a exponential distribution using asymptotic property of maximum likelihood estimators.