(b) Attempt any *two* of the following: [12]

(i) Perform two orthogonal transformations S_1 and S_2 to reduce the given symmetric matrix A to the diagonal form using Jacobi method.

$$A=\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

(ii) Obtain the second iteration matrix A_3 and hence the approximate eigen values of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ by Rutishauser method.

(iii) Determine the largest eigen value and the corresponding eigen vector of the matrix.

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

by Power method. Take initial approximate vector $v^{(0)} = [1,1,1]^T$ and perform three iterations only.

(iv) Find the smallest eigen value in magnitude of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

using two iterations of inverse power method. Take initial approximate vector as $v^{(0)} = [1,1,1]^T$

4. Attempt any *three* of the following: [15]

(i) If 1.414 is used as an approximation to $\sqrt{2}$, then find the absolute error and the relative error.

(ii) Find a root by Secant method correct upto 4 decimal places for the equation $e^{-x} = \sin x$.

(iii) Determine the number of real and complex roots of the polynomial $f(x) = x^4 - 4x^3 + 3x^2 + 4x - 4 = 0$ using Descarte's rule of sign.

(iv) Let P_k be the sum of the moduli of the elements along k^{th} row including the diagonal element a_{kk}. Then show that every eigen value of symmetric matrix A lies inside or on the boundary of atleast one of the circles.

$$|\lambda - a_{kk}| = P_k \ , \quad k=1(1)n$$

(v) Determine the largest eigen value and the corresponding eigen vector of the matrix, $A = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}$ by Power method. Perform three iterations by taking initial approximate vector as $v^{(0)} = [1,1]^T$

(vi) Obtain the first iteration matrix A_2 and hence find approximately the eigen values of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ by Rutishauser method.

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(21/2 Hours) [Total Marks: 75

N.B.: (1) All questions are compulsory.
(2) Figures to the right indicate marks for respective sub-questions.

1. (a) Attempt any *one* of the following [8]

(i) Show that the Newton – Raphson iterative formula applied to the function $f(x) = x^2 - a$, $a > 0$ leads to the iterative formula $x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right)$, $x_0 > 0$ for evaluating $\sqrt{a}$; also show that $x_{k+1} - \sqrt{a} = \frac{1}{2x_k}\left(x_k - \sqrt{a}\right)^2$, hence prove its convergence is of order 2.

(ii) Show that the rate of convergence of Regula Falsi method is linear.

(b) Attempt any *two* of the following: [12]

(i) Consider Newton Raphson method in the form $x_{k+1} = x_k - \beta \frac{f_k}{f'_k}$ .... (I) ,
If α is a multiple root of $f(x) = 0$ with multiplicity m ($m \neq 1$), then determine the value of the parameter β so that the method (I) as given above, has quadratic rate of convergence.

(ii) Using Muller method, solve $x^6 - 7x^4 + 15x^2 - 9 = 0$. Perform one iteration. Take $x_0 = 1.5$, $x_1 = 1.6$, $x_2 = 1.7$.

(iii) Find a positive root of $x^3 - 5x + 3 = 0$, taking $x_0 = 0.5$ using Newton Raphson method. Perform 4 iterations.

(iv) The function $f(x) = x^2 - 3x + 1$, can have 3 possible alternatives for an equivalent fixed point function g(x). Find $g'(x)$ in each case and find the interval on which it is less than 1.

2. (a) Attempt any *one* of the following: [8]

(i) If p_k is an approximation of the root p of the polynomial equation $P_n(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n = 0$, then show that the next approximation to the root using Birge-Vieta method is $p_{k+1} = p_k - \frac{b_n}{c_{n-1}}$, k = 0, 1, 2... where b_k satisfies the recurrence relation $b_k = a_k + p\, b_{k-1}$ with $b_0 = a_0$ and c_k satisfies the recurrence relation $c_k = b_k + pc_{k-1}$ and $c_0 = b_0$.

(ii) Describe Triangularization method for solving numerically a system of linear equations.

(b) Attempt any *two* of the following: [12]

(i) For a matrix define spectral radius, condition number and maximum row sum norm.

(ii) Determine the sturm's sequence of the polynomial equation
$f(x) = x^4 - 4x^3 + 3x^2 + 4x - 4 = 0$.

(iii) Use Birge--Vieta method to determine the deflated polynomial of the equation $x^3 - 11x^2 + 32x - 22 = 0$. Take p =0.5 and perform 3 iterations.

(iv) Find inverse of the following matrix using Cholesky method. A= $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$

3. (a) Attempt any *one* of the following: [8]

(i) Explain Power method to determine the largest eigen value and the corresponding eigen vector of the system $Ax = \lambda x$ where A is a symmetric matrix of order 'n', λ is eigen value of A and x is its corresponding vector.

(ii) Let A be a real symmetric matrix. Using Jacobi method, reduce A to a diagonal matrix by a series of orthogonal transformations $S_1, S_2, \ldots$ in 2×2 subspaces. Let $|a_{ik}|$ be the numerically largest off diagonal element of the matrix S_1^* such that $S_1^{*-1} A S_1^*$ is diagonalized and hence show that $\tan 2\theta = \frac{2a_{ik}}{a_{ii} - a_{kk}}$, $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

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