B. N .BANDODKAR COLLEGE OF SCIENCE, THANE
SEMESTER END EXAMINATION 2013
SEMESTER III USST301

DURATION:2 HOURS

MARKS :60

N.B.: 1 All questions are compulsory.
2 Use of simple calculator is allowed.

Q.1 (a) **Attempt any ONE**

(1) Moment Generating Function. (m.g.f.) and Cumulant Generating Function. (c.g.f.) of a random variable (r.v.) X. are $M_X(t)$ and $K_X(t)$ respectively. Show that i)$M_{aX+b}(t)=e^{bt}M_X(at)$ ii) $K_{aX+b}(t)=at+K_X(bt)$,where a,b are constants. 7

2) i)A continuous r.v X has p.d.f f(x).Y=g(X) is function of X. Show that p.d.f of Y is given by $h(y)=f(g^{-1}(y))\left|\frac{dg^{-1}(y)}{dy}\right|$ 7

ii)A r.v X follows uniform distribution over the range (0,1) . Obtain p.d.f of $Y=\frac{X}{1+X}$

(b) **Attempt any ONE**

(1) i)$X_1, X_2, \ldots, X_n$ are independent random variables (r.v. s) ,with m.g. f. $M_{X_i}(t)$,i=1,2,....n..respectively, then 8

$$M_{\sum_{i=1}^{n} X_i}(t)=\prod_{i=1}^{n} M_{X_i}(t)$$

ii)If $M_{X_i}(t)=(1-\frac{t}{\theta})^{-1}$ $\left|\frac{t}{\theta}\right|<1$, Find the m.g. f of $Y=\sum_{i=1}^{n} X_i$ iii) Find mean and variance of Y.

(2) Define r[th] cumulantof a r.v . Derive the relations between first four central moments and cumulants. 8

Q.2 (a) **Attempt any ONE**

(1) A r.v. X follows Binomial distribution with parameters (n, p), with usual notations show that $\mu_{r+1} = pq\left(\frac{d\mu_r}{dp}+n\, r\mu_{r-1}\right)$.Hence find β_1, β_2. 7

(2) A r.v X follows Negative Binomial distribution with parameters (k, p), obtain its mean and variance. 7

(b) **Attempt any ONE**

(1) A r.v. X follows Poisson distribution with parameter λ. Obtain expression for its cumulant generating function . Hence find its mean and variance, β_1, β_2.Further show that $\sqrt{\beta_1}(\beta_2-3)\mu_1'\sigma=1$ 8

(2) i)Find m. g. f of geometric variate with parameter p . 8

P.T.O

ii)Hence show that if Y is sum of k independent identically distributed (i.i.d) geometric variates with parameter p , has Negative Binomial with parameters k and p . ii)Write p .m. f of Y.

Q.3 (a) **Attempt any ONE**

(1) Define i)Bivariate continous r.v.s.(X,Y),ii) Joint p.d.f.of (X,Y) , iii) Marginal p.d.f of X iv) Marginal p.d.f of Y, v) Conditional p.d.f.X given Y=y, vi) Conditional p.d.f.Y given X=x, vii) Independence of r.v.s (X,Y). 7

(2) Random variable X and Y are independently identically distributed each having an exponential distribution with parameter λ. Obtain joint p.d.f. of U= X+Y , V = X/Y. Are they independent? 7

(b) **Attempt any ONE**

(1) Define Conditional expectation and conditional variance of X/Y=y. Further with usual notations show that 8

i) E(E(X/Y=y))=E(X)

ii) E(V(X/Y=y))+V(E(X/Y=y)) =V(X)

(2) The r.v s (X,Y) have following joint p.d.f . 8

$$f(x,y)=\begin{cases} 6x\,y^2 & 0 \le x, y \le 1 \\ 0 & \textit{otherwise} \end{cases}$$

i)Obtain Marginal p.d.f s of X and Y ii)Verify independence of the r.vs X,Y, iii)Find P[X< 0.4 ,Y <0.4)

Q.4 **Attempt any THREE**

(1) Define characteristic function. State its properties. 5

(2) A r.v X has Uniform distribution over (-1,1). Obtain p.d.f of Y=X^2. 5

(3) Obtain p.m.f. of truncated Binomial distribution truncated at zero. Also obtain its mean. 5

(4) Show that under certain conditions to be stated by you the hypergeometric distribution can be approximated to binomial distribution . 5

(5) With usual notations show that 5

i) E(aX+bY)= aE(X)+bE(Y)

ii) V(aX+bY)= a^2V(X)+b^2V(Y)+2ab COV(X,Y)

(6) Define correlation coefficient. State and prove its any one property. 5

****