(21/2 Hours) Total Marks: 75

N.B.: (1) All questions are compulsory.
(2) Figures to the right indicate marks for respective sub questions.

1. (a) Attempt any ***one*** of the following: **[8]**

(i) Show that the rate of convergence of Secant method is 1.618.

(ii) Derive the Regula-Falsi iteration formula

$$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k), \; f(x_k)\, f(x_{k-1}) < 0$$

for k=1,2... to find the root of the continuous differentiable function f(x).

(b) Attempt any ***two*** of the following: **[12]**

(i) Find the smallest positive root correct using false position method for the equation $x^3 - 5x + 1 = 0$. Perform two iterations

(ii) The equation $f(x) = x^4 - x - 10 = 0$ has a root in (1, 2). Perform two iterations of Secant method.

(iii) Apply Newton-Raphson method to determine a root of the equation $x^3 = 1 - x^2$ such that $|f(x^*)| < 10^{-4}$, where * is the approximation to the root. Take initial approximation, $x_0 = 1$.

(iv) Perform one iteration using Muller method for the equation
$x^3 - 5x + 1 = 0$, $x_0 = 0$, $x_1 = 0.5$, $x_2 = 1$.

2. (a) Attempt any ***one*** of the following: **[8]**

(i) If p_k is an approximation of the root p of the polynomial equation $P_n(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n = 0$, then show that the next approximation to the root using Birge-Vieta method is $p_{k+1} = p_k - \frac{b_n}{c_{n-1}}$, $k = 0, 1, 2\ldots$ where b_k satisfies the recurrence relation $b_k = a_k + p\, b_{k-1}$ with $b_0 = a_0$ and c_k satisfies the recurrence relation $c_k = b_k + p\, c_{k-1}$ and $c_0 = b_0$.

(ii) Derive Chebyshev iteration formula

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} - \frac{1}{2}\left\{\frac{f(x_k)}{f'(x_k)}\right\}^2 \left\{\frac{f''(x_k)}{f'(x_k)}\right\}, \; k = 1, 2, 3\ldots$$

for finding the roots of the polynomial $f(x) = a_0x^2 + a_1x + a_2$ where a_0, a_1 and a_2 are constants.

(b) Attempt any ***two*** of the following: **[12]**

(i) Solve $x^3 - 2 = 0$, using Multipoint Iteration Method, correct to four places of decimals with the initial approximation as $x_0 = 1$. Perform two literations.

(ii) Use Birge-Vieta method to find the smallest positive root of the equation
$x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$, $p_0 = 1.5$. Perform two iterations.

(iii) Perform two iterations using Chebyshev method to find an approx. value of 1/7.
Take $x_0 = 0.1$

(iv) Find the real and complex roots of the polynomial $2x^3-x^2+2x-1=0$ using Sturm sequences.

3. (a) **Attempt any *one* of the following:** [8]

(i) Describe Gauss- Seidel iterative method for solving a system of linear equations numerically. Give sufficient conditions for convergence of the process.

(ii) Describe Jacobi iterative method to find the eigenvalues of a real symmetric matrix,A.

(b) **Attempt any *two* of the following:** [12]

(i) Solve the following a linear system of equations using Crout's method:

$x + 2y + z = 4$, $2x + 5y + 4z = 11$ and $x + 4y + 6z = 11$.

(ii) Use Jacobi iteration method to solve a system of equations:

$10x + y - z = 7$, $x + 10y + z = 15$, $x + y - 10z = 7$. Take $x^{(0)} = y^{(0)} = z^{(0)} = 0$. Perform two iterations.

(iii) Use Jacobi iteration method, to find the eigenvalues and eigenvectors of a matrix, $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

(iv) Determine the smallest eigenvalue in magnitude and corresponding eigenvector for a matrix, $A = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}$ using power method. Perform two iterations by taking an initial approximation to the eigenvector as $v^{(0)} = [2, -3]^t$.

4. **Attempt any *three* of the following:** [15]

(a) Find a root of $x - \cos x = 0$ using Secant method by taking $x_0 = 0$ and $x_1 = 1$. Perform two iterations.

(b) Show that the equation $f(x) = x^3 - x^2 - x + 1$ has a double root at $x = 1$. Perform two iterations of Newton-Raphson method by taking $x_0 = 0.5$

(c) Perform one iteration of Muller method to find a root of an equation $xe^x - \cos x = 0$. Take $x_0 = -1$, $x_1 = 0$, $x_2 = 1$ and perform one iteration.

(d) Use the Birge-Vieta method to find a real root of $P(x) = x^4 - 2x^3 - 3x^2 + 2x + 2$. Take $p_0 = 2$ and perform two iterations.

(e) Solve a systems of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ using Cholesky method.

(f) For the following system of equations: $2x - y = 7$, $-x + 2y - z = 1$ and $-y + 2z = 1$ Obtain the Gauss – Seidel iteration scheme in the matrix form starting with $x^{(0)} = [0, 0, 0]^t$. Iterate two times.

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