(3 Hours) **[Total Marks: 100]**

Note: (*i*) All questions are compulsory.
(*ii*)Figures to the right indicate marks for respective parts.

Q.1 Choose correct alternative in each of the following **(20)**

i. The order and degree of differential equation $\sqrt{\left(\frac{d^3y}{dx^3}\right)^2 + x + 4\left(\frac{dy}{dx}\right)^3} = 3x^3$ is
(a) 1 and 3 (b) 2 and 1
(c) 2 and 2 (d) 3 and 2

ii. Which of the following equations is exact?
(a) $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$ (b) $x^2y^2dx + (y^2 + x)dy = 0$
(c) $(3x + y)^2dx + x^2dy = 0$ (d) None of these.

iii. Which of the following substitution converts non-homogeneous differential equation $(x + y)dy + (5x - 3y + 8)dx = 0$ into a homogeneous differential equation?
(a) $x = X - 1, y = Y + 1$ (b) $x = X + 1, y = Y + 1$
(c) $x = X + 1, y = Y - 1$ (d) $x = X - 1, y = Y - 1$

iv. Which of the following functions are not linearly independent on $[1, 2]$?
(a) $y = \sin x$ and $y = \cos x$ (b) $y = x^2$ and $y = x^3$
(c) $y = \log x^2$ and $y = \log x^3$ (d) None of these

v. One of the solutions of the differential equation $y'' - y = 0$ is
(a) $y = k$, where $k \in \mathbb{R}\backslash\{0\}$ (b) $y = \sin x$
(c) $y = e^x$ (d) None of these

vi. The Wronskian of $y_1 = \sin x$ and $y_2 = \cos x$ is
(a) 1 (b) 0
(c) -1 (d) None of these

vii. The roots of the auxiliary equation of the differential equation $y'' - 4y' + 3y = 0$ are
(a) 1 and 4 (b) 3 and 4
(c) 1 and 3 (d) None of these

viii. The auxiliary equation of the system of homogeneous linear differential equations $\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y \end{cases}$ is

(a) $m^2 - (a_1 + b_2)m + a_1b_2 - a_2b_1 = 0$

(b) $m^2 - (a_2 + b_1)m + a_1b_2 - a_2b_1 = 0$

(c) $m^2 + (a_1 + b_2)m + a_1b_2 - a_2b_1 = 0$

(d) $m^2 - (a_2 + b_1)m + a_2b_1 - a_1b_2 = 0$

ix. One of the solutions of the system $\begin{cases} \frac{dx}{dt} = y - x \\ \frac{dy}{dt} = 3x + y \end{cases}$ is

(a) $\begin{cases} x = 3e^{2t} \\ y = e^{2t} \end{cases}$ (b) $\begin{cases} x = e^{2t} \\ y = 3e^{2t} \end{cases}$

(c) $\begin{cases} x = e^{t} \\ y = 3e^{t} \end{cases}$ (d) None of these

x. The auxiliary equation of the system $\begin{cases} \frac{dx}{dt} = 3x + 2y \\ \frac{dy}{dt} = -5x + y \end{cases}$ has

(a) Real and distinct roots (b) Roots which are complex conjugates

(c) Real and repeated roots (d) Does not have any roots

Q2. Attempt any **ONE** question from the following: **(08)**

a) i. Show that the general solution of the linear differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is $y = e^{-\int P\,dx}\left[\int Q \cdot e^{\int P\,dx}dx + c\right]$.
Hence solve $(x^2 + 1)\frac{dy}{dx} + 2xy = 4x^2$.

ii. Show that a necessary and sufficient condition for the differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ where the functions M, N possess continuous first order partial derivatives on the domain under consideration.

Q.2 Attempt any **TWO** questions from the following: **(12)**

b) i. Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.

ii. Using Bernoulli's method find the general solution of $x\frac{dy}{dx} + y + 2x^6y^4 = 0$.

iii. Find the family of orthogonal trajectories of the family of parabolas $y^2 = 4ax$ where a is a parameter.

iv. Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$

Q3. Attempt any **ONE** question from the following: **(08)**

a) i. Let y_1 be a non-zero solution of the homogeneous differential equation $y'' + P(x)y' + Q(x) = 0$. Assuming $y_2 = vy_1$, show that another solution (which is linearly independent of y_1) of the same differential equation can be found out, where $v = \int \frac{1}{y_1^2} e^{-\int P dx} dx$.

If $y_1 = x$ is one solution of $x^2y'' + xy' - y = 0$, then find another solution of the same, which is linearly independent of y_1.

ii. Explain the method of variation of parameters to find the general solution of a non-homogeneous differential equation, when the general solution of the corresponding homogeneous differential equation is known.

Q3. Attempt any **TWO** questions from the following: **(12)**

b) i. Show that any linear combination of two solutions of the homogeneous equation $y'' + P(x)y' + Q(x)y = 0$ is also a solution of the same. Hence or otherwise, show that $e \sin x + \pi \cos x$ is a solution of $y'' + y = 0$.

ii. Verify that $y_1 = x^2$ is one solution of $x^2y'' + xy' - 4y = 0$, and find the general solution of the same.

iii. By using the method of undetermined coefficients, find the general solution of the differential equation, $y'' - y' - 6y = 20e^{-2x}$.

iv. By using the method of variation of parameters, find the general solution of the differential equation: $y'' + 4y = \tan 2x$.

Q4. Attempt any **ONE** question from the following: **(08)**

a) i. Define Wronskian $W(t)$ of the two solutions

$\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$

where $a_1(t), a_2(t), b_1(t), b_2(t)$ are continuous functions on $[a, b]$. Show that their Wronskian is either identically zero or nowhere zero on $[a, b]$.

ii. Find the general solution of system $\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y \end{cases}$ where

a_1, a_2, b_1 *and* b_2 are constants when the roots of auxiliary equation are real and distinct.