Duration: 2 Hours **Total Marks: 60**

NB:

1. **All questions are compulsory.**
2. **Figures to the right indicate full marks.**
3. **Use of scientific calculators is allowed.**

Q. 1 A) Answer ***any one*** of the following: **06**

i) State and derive the Newton Raphson method of finding root of an equation $f(x) = 0$.

ii) Describe the method of fixed-point iteration method. Give an example to show that the method need not converge always.

B) Answer ***any two*** of the following: **10**

i) Perform two iterations of Newton Raphson method to find the approximate value of $(17)^{\frac{1}{3}}$ with $x_0 = 2$.

ii) Perform 2 iterations of the bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$.

iii) Using fixed-point iteration method, find a root of $xe^x = 1$ that lies between 0 and 1, correct upto two decimals.

iv) Using two iterations of Regula-Falsi method, find approximate root of the equation $\cos x - xe^x = 0$.

Q. 2 A) Answer ***any one*** of the following: **06**

i) Derive Newton's Backward Difference Interpolating polynomial.

ii) Derive the Lagrange quadratic interpolating polynomial.

B) Answer ***any two*** of the following: **10**

i) Obtain linear least squares polynomial approximation to the given data:

x	0.2	0.4	0.6	0.8	1
$f(x)$	0.108	0.164	0.316	0.612	1.1

ii) With usual notations, prove that $\Delta^4 y_0 = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$ and hence find the missing term in the following table:

x	0	1	2	3	4
$f(x)$	1	3	9	—	81

iii) Consider the following data:

x	0.10	0.15	0.20	0.25	0.30
$y = \tan x$	0.1003	0.1511	0.2027	0.2553	0.3093

Using Gregory-Newton's backward difference interpolation, find $\tan(0.26)$.

iv) Given $f(0) = 1, f(1) = 3$ and $f(3) = 55$. Find the unique polynomial of degree 2 which fits the given data using Lagrange Interpolation.

Q. 3 A) Answer ***any one*** of the following: **06**

i) Explain the following methods:

a) LU decomposition method b) Rutishauser method

ii) Derive the Jacobi's iteration method for finding solution of a system of linear equations.

B) Answer ***any two*** of the following: **10**

i) Decompose the matrix $\begin{pmatrix} 2 & -6 & 10 \\ 1 & 5 & 1 \\ -1 & 15 & -5 \end{pmatrix}$ using Dolittle's LU decomposition method.

ii) Solve the following system using 2 iterations of Jacobi's iteration method:

$$-x_2 + 2x_3 = 1$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$2x_1 - x_2 = 7$$

iii) Find eigenvalues of $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ using 2 iterations of Rutishauser method.

iv) Solve the following system using 2 iterations of Gauss Seidel iteration method:

$$\begin{pmatrix} 5 & 1 & 2 \\ 3 & 4 & -1 \\ 2 & -3 & 5 \end{pmatrix} X = \begin{pmatrix} 2 \\ -2 \\ 10 \end{pmatrix}$$

Q. 4 Answer ***any two*** of the following: **12**

a) Perform 3 iterations of the secant method to find root of the equation $x - e^{-x} = 0$.

b) Explain the linear least squares approximation method for discrete data and derive the normal equations for it.

c) Evaluate $\int_0^1 \frac{1}{1+x} dx$ using i) Trapezoidal rule and ii) Simpson`s $1/3^{rd}$ Rule

d) Find all the eigenvalues and the corresponding eigenvectors of $A = \begin{pmatrix} 1 & \sqrt{3} & 4 \\ \sqrt{3} & 5 & \sqrt{3} \\ 4 & \sqrt{3} & 1 \end{pmatrix}$

using 2 iterations of Jacobi`s method.

---END--