Duration : $2\frac{1}{2}$ hours **Total Marks : 75**

N.B. (1) All questions are compulsory.

Q.1 a) Attempt any ONE of the following. **(8 M)**

i. Prove that the general solution of the Lagrange's equation in three variables is an arbitrary differentiable function $F(u,v) = 0$. Where $u(x, y, z) = c_1$ & $v(x, y, z) = c_2$ are linearly independent solutions of auxiliary equations.

ii. Prove that the necessary & sufficient condition that the Pfaffian differential equation $P(x, y, z)\,dx + Q(x, y, z)\,dy + R(x, y, z)\,dz = 0$ be integrable is that $\bar{X} \cdot \text{curl}\,(\bar{X}) = 0$ Where $\bar{X} = (P, Q, R)$.

b) Attempt any TWO of the following **(12 M)**

i. Show that $y(x + z) = c(y + z)$ is an integral of $(y^2 + yz)\,dx + (xz + z^2)\,dy + (y^2 - xy)\,dz = 0$.

ii. Define the singular integral of the pde $f(x, y, z, p, q) = 0$ & prove that singular integral is also a solution.

iii. Find the general solution of $xp + yq = z$.

Q.2 a) Attempt any ONE of the following. **(8 M)**

i. Let $f(x, y, z, p, q) = 0$ & $g(x, y, z, p, q) = 0$ are two given first order pde. State & Prove the necessary & sufficient condition for the integrability of $dz = p\,dx + q\,dy$

ii. Define the compatiblity of the equations $f(x,y,z,p,q) = 0$ & $g(x,y,z,p,q) = 0$ on the domain D.
Hence show that $xp - yq - x = 0$ & $x^2p + q - xz = 0$ are compatible .
Show by examples that if two pde are compatible then it is not necessary that they have every solution to be common.

b) Attempt any TWO of the following. **(12 M)**

i. Find the complete integral of $p^2 + q^2 = (x + y)$.

ii. Find a complete integral of $xpq + yq^2 = 1$ by Charpit's method.

iii. Solve $(u_z + z)\,z = u_x^2 + u_y^2$ by Jacobi's method.

P.T.O.

Q. 3 a) Attempt any ONE of the following. **(8M)**

i. Consider the first order quasi – linear pde
$P(x, y, z)\, z_x + Q(x, y, z)\, z_y = R(x, y, z)$

Where P, Q, R have continous partial derivatives with respect to x, y, z & they do not vanish simultaneously . Let the value $z = z_0(s)$ be prescribed along the intial curve C : $x = x_0(s)$, $y = y_0(s)$, x_0, y_0 & z_0 being continuously differentiable functions. Further for $a \le s \le b$,
if $\frac{dy_0}{ds} P(x_0(s), y_0(s), z_0(s)) - \frac{dx_0}{ds} Q(x_0(s), y_0(s), z_0(s)) \neq 0$,

prove that there exist a unique solution z(x, y) defined in some neighbourhood of intial curve C which satisfy the pde & $z(x_0(s), y_0(s)) = z_0(s)$.

ii. If an element $(x_0, y_0, z_0, p_0, q_0)$ is common to both an integral surface $z = z(x, y)$ & a characteristic strip. Prove that the corresponding characteristic curve lies completely on the integral surface $z = z(x, y)$.

b) Attempt any TWO of the following. **(12 M)**

i. Solve the intial value problem
For the equation $z_x - z z_y + z = 0$ containing the intial data curve
C : $x_0 = 0$, $y_0 = s$, $z_0 = -2s$, For all $s \in \mathbb{R}$

ii. Find the characteristic strip through
C: $x_0 = s$, $y_0 = 0$, $z_0 = s$, of the equation $z_x z_y - xy = 0$.

iii. Find the integral surface of $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ passing through $x_0(s) = 1$, $y_0(s) = 0$, $z_0(s) = s$.

Q. 4 Attempt any THREE of the following. **(15 M)**

i. Find pde whoes complete integral is $z = ax + by + a^2 + b^2$. Find a singular integral of the pde by eliminating a & b from complete integral.

ii. Find the general integral of $x(y - z)p + y(z - x)q = z(x - y)$.

iii. Solve $z^3 = pqxy$ by Jacobi's method.

iv. Find a Complete integral of $zpq = p + q$ by Charpit's method.

v. Find by the method of characteristics the integral surface of $pq - z = 0$ through the curve C : $z = y^2$, in YZ plane.

vi. Define the Monge cone. Find the Monge cone for the pde $p^2 + q^2 = 1$ at (0, 0, 0).

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