B. N. BANDODKAR COLLEGE OF SCIENCE, THANE

THIRD SEMESTER END EXAMINATION, OCTOBER 2014

USMT 301

DURATION: 2 HOURS 30 MINS **MARKS: 75**

Instructions to the candidates:

1. **All questions are compulsory.**
2. **Figures to the right indicate marks.**
3. **Calculators are not allowed.**

Q.1 (a) Attempt any one of the following:

(i) If a < b and b < c then prove that a < c. 8
If a > 0 and b > 0 then prove that $\sqrt{ab} \leq \frac{a+b}{2}$.

(ii) State and prove Hausdorff's property in IR. 8
Find disjoint neighbourhood of 0.01 and 0.001.

(b) Attempt any three of the following:

(i) Prove that the set of all natural numbers is closed in IR. 4
(ii) Prove that arbitrary intersection of closed sets is closed in IR. 4
(iii) Prove that if a set contains all its limit points then it is a closed set. 4
(iv) Prove that union of two open set in IR is open in IR. 4
(v) State and prove Archimedian property in IR. 4
(vi) If x > 0 then prove that $(1+x)^n \geq 1 + nx \; \forall n \; \in IN$. 4

Q.2 (a) Attempt any one of the following:

(i) If $(x_n) \to p$ and $(y_n) \to q$ then prove that $(x_n + y_n) \to p + q$. 8
If $(x_n) \to p$ and $(y_n) \to q$ then prove that $(x_n y_n) \to pq$.

(ii) State and prove sandwich Theorem of a sequence. And use it to prove that the 8
sequence $\left(n^{\frac{1}{n}}\right) \to 1 \; for \; n \in IN$.

(b) Attempt any three of the following:

(i) Prove that the sequence $\left(\frac{1}{n}\right)$ *is a* convergent sequence. 4

(ii) Prove that every Cauchy sequence is convergent. 4

(iii) Find limit superior and limit inferior of the sequence $\frac{1}{n+1}$. 4

(iv) If $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$ then prove that $\lim_{x\to a} f(x) + g(x) = l + m$. 4

(v) Prove that the sequence $\left(\frac{1}{n+1}\right)$ is a Cauchy sequence. 4

(vi) If $f : IR \rightarrow IR$ is given by 4

$$f(x) = 1 \; if \; x \in Q$$
$$f(x) = -1 \; if \; x \in IR \setminus Q$$

Prove that f is not continuous.

Q.3 (a) Attempt any one of the following:

(i) Discuss the convergence of series $\sum \frac{1}{n^p}, p > 1$. 8

Hence prove that the series $\sum \frac{n^2}{n^4+n^3+1}$ converges.

(ii) State and prove limit form of root test. 8

Discuss the convergence of the series $\sum \frac{1}{n^n}$.

(b) Attempt any three of the following:

(i) State and prove comparison test for convergence of series. 4

(ii) State and prove Cauchy criterion for convergence of series. 4

(iii) If $\sum x_n$ converges then prove that $\sum (\alpha x_n)$ converges for $\alpha \in IR$. 4

(iv) Find radius of convergence of $\sum \frac{x^n}{2n+1}$. 4

(v) Find Fourier series of f(x) = x for x $\in [-\pi, \pi]$. 4

(vi) If $\sum x_n$ converges then prove that lim $x_n = 0$. Is the converse true? Justify. 4

Q.4 Attempt any three of the following:

(i) If $A \subseteq B$ then prove that $L(A \cup B) = L(A) \cup L(B)$. 5

(ii) Prove that the set $\{x \in IR | x^2 + 3x + 2 < 0\}$ is bounded set. 5

(iii) If a sequence converges then prove that it converges to unique limit. 5

(iv) Prove that the sequence $\left(\frac{3n+2}{2n+1}\right) \rightarrow \frac{3}{2}$. 5

(v) Discuss the convergence of the series $\sum \frac{n^2}{n^5+2}$. 5

(vi) Discuss the convergence of the series $\sum \frac{2n}{n(n+1)(n+2)}$. 5

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