Duration: 2hrs **Marks: 60**

Notes: 1) All questions are compulsory and carry equal marks.
2) Figures to right indicate marks.
3) Use of calculator is allowed.

Q.1 a) **i)** Define Moment generating function (m. g. f.) of any random variable. **(8)**
ii) If $X_1, X_2, \ldots\ldots\ldots\ldots, X_n$ are independent random variables (r. v. s) with m.g. f. $M_{Xi}(t)$, i=1,2,…….n respectively, then show that $M_{\sum_{i=1}^{n} xi}(t) = \prod_{i=1}^{n} Mxi(t)$
iii) X_i follows Binomial Distribution then find m. g. f. of $Y=\sum_{i=1}^{n} Xi$ where X_i are independent r. v. s.

OR

Q.1 a) **i)** Define characteristic function (Ch.f.). State its properties. **(8)**
ii) If $\phi_x(t) = 4(3-e^{it})^{-2}$, obtain mean and variance.

Q.1 b) A random variable X follows Binomial Distribution with parameters n and p, Derive mode of Binomial Distribution with parameters n and p. **(8)**

OR

Q.1 b) With usual notations **(8)**

i) If $K_x(t) = m(e^t - 1)$, obtain its first four moments hence β_1 and β_2

ii) If $Mx(t) = (1-2t)^{-1}$, obtain expression for μ_r' .Hence β_1 and β_2

Q.2 c) **i.** A random variable X follows Geometric Distribution with parameter p, with usual notations show that $P[\, X \geq r+s \,/\, X \geq s \,] = P[\, X \geq r \,]$ **(8)**
ii. Obtain mean and variance of Hyper-geometric Distribution.

OR

Q.2 c) A random variable X follows Negative Binomial Distribution with parameter k,p with usual notations show that **(8)**

$$\mu_{r+1} = q\left[\frac{d\mu}{dp} + \frac{rk}{p^2}\mu_{r-1}\right]$$

hence find β_1 and β_2

Q.2 d) If X_1 and X_2 are two independent Binomial variates such that X_1 follows Binomial with parameters n_1 and p and X_2 follows Binomial with parameters n_2 and p. Show that the conditional distribution of X_1 given X_1+X_2 follows Hyper-geometric distribution. **(8)**

OR

Q.2 d) **i)** Show that under certain conditions to be stated the Negative Binomial Distribution can be approximated to Poisson Distribution. **(8)**
ii) State and prove additive property of Poisson variates .

Q.3 e) If X and Y are any 2 r. v.s. with $E(X) = \mu_x$ and $V(X) = \sigma_x^2$; $E(Y) = \mu_y$ & $V(Y) = \sigma_y^2$. ρ is the correlation coefficient between x and y . Then find value of k **(8)**
Where $U = x + \frac{\sigma_x}{\sigma_y} Y$ and $V = Y + kX$
And U and V are uncorrelated.

OR

Q.3 e) If f(x,y) = kxy 0<x<y<1 be bivariate p. d. f. Obtain the value of k. Also obtain marginal densities of X and Y. Write down theexpression for conditional p. d. f. of Y given X=x. Check whether X and Y are independent. **(8)**

Q.3 f) **i.** Define conditional expectation and conditional variance of X/Y=y. **(8)**

ii. With usual notations show that

$E(E(X/Y=y))=E(X)$

$E(V(X/Y=y))+V(E(X/Y=y))=V(X)$

$$\rho = \frac{V(X)+V(Y)-V(X-Y)}{2\sqrt{V(X)*V(Y)}}$$

OR

Q.3 f) **i.** Find value of k so that U and V are uncorrelated. **(8)**

$U = X + kY$ and $V = X + (\sigma_x / \sigma_y)Y$

ii. With usual notations show that

a) $E(aX+bY) = aE(X) + bE(Y)$

b) $V(aX+bY) = a^2V(X) + b^2V(Y) + 2abCov(X,Y)$.

Q.4 **Attempt any 3** **(12)**

1. Obtain p. m. f. of truncated Poisson Distribution truncated at zero. Also obtain its mean.
2. If X is Uniform discrete random variable. Find mgf of Y=X+k hence or otherwise find its mean and variance.
3. If random variables X and Y are two independent Geometric variates with parameters p then show that $P[X=Y] = p/(1+q)$
4. Define the following:
   - i. Marginal p. d. f. of X
   - ii. Conditional p. d. f. X given Y=y
   - iii. Independence of r. v. s. (X,Y)
   - iv. Uncorrelated r. v. s.
5. Let X is a random variable having Exponential Distribution with mean 1. Find its mgf and cgf.

*****ALL THE BEST*****