

Q. 1

Attempt any four of the following

a. Given that, $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & -1 \end{bmatrix}$

Verify that, i. $(AB)^* = B^*A^*$

b. State whether the following vectors are linearly dependent or are independent. If dependent find the relation between them $x_1 = (1, 2, 3)^T$, $x_2 = (3, -2, 1)^T$, $x_3 = (1, -6, -5)^T$

c. Find the characteristic values and characteristic vector of the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

d. Simplify,

$$\frac{(\cos 3\theta + i \sin 3\theta)^4 (\cos 5\theta - i \sin 5\theta)^{4/5}}{(\cos^3 \theta + i \sin^3 \theta)^{2/9} (\cos^4 \theta - i \sin^4 \theta)^{10}}$$
 using De-mover's theorem

e. Find z_1, z_2 and z_1/z_2 where $z_1 = 1 - 2i$ and $z_2 = 3 - 4i$

f. Find the values of $\text{Log}(-5)$ and $\log(1+i)$

g. Solve the following homogeneous differential equation $(x^2y)dx - (x^3 + y^3)dy = 0$

h. Solve $(2D^2 + 5D - 12)y = 0$

Q. 2

Attempt any four of the following

a. Find the complimentary function of the equation $(D^2 + 4D + 3)y = e^x$

b. Find the particular integral of the equation $(D^3 - 2D^2 - 5D + 6)y = e^{4x}$

c. State and prove the second shifting theorem of Laplace transformation.

d. Find $L\{F(t)\}$ for

i. $\frac{t}{2a} \sin at$ ii. $t^2 \cos at$

e. Find the Laplace transformation of $\sin at$ and hence show that,

$$\int_0^\infty \frac{\sin t}{t} \cdot dt = \frac{\pi}{2}$$

f. Verify the convolution theorem for the pair of function, $f_1 = t$, $f_2 = e^{at}$.

g. Find the inverse Laplace transformation of the following function

$$\frac{s^2}{(s^2 + a^2)^2}$$

h. Obtain the inverse Laplace transformation of the following functions.

$$\frac{s}{(s^2 + a^2)^2}$$

Q. 3

Attempt any four of the following

a. Find the Laplace transformation of direct delta function.

b. Define periodic function also find Laplace transformation of the periodic function

c. Evaluate, $\int_0^1 \int_0^2 e^{x+y} dx dy$

d. Evaluate, $\int_0^2 \int_0^{1/4} xy \cdot dx \cdot dy$

e. Show that $\int_0^a x^6 (a^4 - x^4)^{1/4} dx = \frac{\sqrt{2}}{128} a^8 \pi$

f. Prove that the beta function is symmetric.

i.e. $\beta(m, n) = \beta(n, m)$

- g. Find $\int_0^1 \int_0^2 \int_0^3 dz \cdot dy \cdot dx$
- h. Change the order of integration in the following integral. $\int_0^1 \int_0^2 \sqrt{1-x^2} dy \cdot dx$ where the region formed by $y=x$, $x=1$

Q. 4

Attempt any Three of the following

12

- a. Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ satisfy its characteristic equation and hence determine A^{-1} .

- b. Explain the properties of the matrices.

- c. Solve $x(1-x^2)\frac{dy}{dx} + (2x^2 - 1)y = x^3$

- d. Solve using variable separable form $\frac{dy}{dx} = e^{x-2y}$

- e. Obtained a Laplace transformation $\frac{d^2y}{dt^2} - 3\frac{dy}{dx} + 5y$, given $y(0)=2$ and $y'(0)=-4$

- f. Evaluate $\int_0^1 \int_0^{x^2} (x + 2y^2) dy \cdot dx$

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