

Duration – 2 Hrs

Max. Marks 60

N.B. All questions are compulsory.

- Q.1** (a) Solve : $|8 - 3x| \leq 7$. (3)
- (b) **Attempt any three of the following.**
- (i) Sketch the graph of $f(x) = x^2 + 5x + 6$. (4)
- (ii) Use $\epsilon - \delta$ definition to prove that $\lim_{x \rightarrow 1} 5x + 3 = 8$. (4)
- (iii) State and prove Sandwich theorem. (4)
- (iv) Let $G : \mathbb{R} \rightarrow \mathbb{R}$ is defined as (4)
- $$G(x) = x \text{ if } x < -2$$
- $$= bx^2 \text{ if } x \geq -2.$$
- Find the value of b for which G is continuous.
- (v) Prove that $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$. (4)
- Q.2** (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \sin x$. Find derivative of f at $x = a$. (3)
- (b) **Attempt any three of the following.**
- (i) Find $\frac{dy}{dx}$ if $(3xy + 7)^2 = 6xy$. (4)
- (ii) Prove that every differentiable function is continuous. (4)
- (iii) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \sqrt{x}$, Find $\frac{d}{dx} f^{-1}(x)$. (4)
- (iv) Find n th derivative of $f(x) = x^3 e^{4x}$. (4)
- (v) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x \tan(2x + 7)$. Find derivative of f(x) using chain rule. (4)
- Q.3** (a) Find $\lim_{x \rightarrow 0} \frac{x - \log(1+x)}{1 - \cos x}$. (3)
- (b) **Attempt any three of the following.**
- (i) Verify Rolle's theorem for $f(x) = x^2 + 8x + 15$, $x \in [-5, -3]$. (4)
- (ii) Prove that $f(x) = x^2 + 4x + 5$ is increasing. (4)
- (iii) State and prove Lagrange's Mean Value Theorem. (4)
- (iv) Find Taylor's expansion of $f(x) = \sin x$. (4)
- (v) Find local Maxima / Minima of $f(x) = 4x^3 - x^4$. (4)
- Q.4** (a) Find nth derivative of $f(x) = \log(3x + 4)$. (3)
- (b) **Attempt any three of the following.**
- (i) Sketch the graph of $f(x) = -2x^3 + 6x^2 - 3$. (4)
- (ii) Prove that $f(x) = |x|$ is continuous for all values of $x \in \mathbb{R}$. (4)
- (iii) Define Ceiling and Flooring function. (4)
- (iv) Prove that $|a| < b$ if and only if $-b < a < b$. (4)
- (v) State and prove Cauchy's Mean value Theorem. (4)
