M.Sc II Sem III Exam
2019
19/06/19

Paper / Subject Code: 94937 / Statistics: Stochastic Processes - I & Survival Analysis - I

[Total marks: 60

(03 hours)

N. B.:

1. Attempt any two questions from question numbers 1, 2, 3 and any two questions from question numbers 4, 5, 6.
2. Figures to the right indicate full marks.
3. Simple non-programmable calculator is allowed.

Q.1 (a) State the postulates of a Poisson process $\{N(t)\}$. (10)

i. Obtain $corr\{N(t), N(t+s)\}$.

ii. Justify $\dfrac{N(t)}{t}$ may be used as a reasonable estimate of the mean rate λ of the process $\{N(t)\}$.

iii. How is exponential distribution related to Poisson process? Give justification.

(b) Describe Poisson cluster process with assumptions. Obtain probability generating function of total number of occurrences in an interval of length t. (05)

Q.2 (a) For a discrete time stochastic process, define the terms, (08)

i. Stationary

ii. Weakly stationary

iii. Markov property.

Let $X_n = Y_1 + Y_2 + \cdots + Y_n$ be a simple random walk with $P[Y_n = 1] = p$, $P[Y_n = -1] = 1 - p$, $n = 1, 2, \cdots$. Explain by giving reasons which of the above properties are satisfied by X_n.

(b) Obtain the transition density matrix of a Poisson process and write Kolmogorov forward equations. (07)

Q.3 (a) Describe LCG method of generating random observations from $U(0, 1)$. Why are they called Pseudo random numbers? (05)

(b) Explain how will you generate random observations from the Laplace distribution, (05)

$$f(x) = \frac{1}{2}\exp^{-|x|} \quad ; \quad -\infty < x < \infty$$

(c) Let X be a random variable with $P[X = i] = p(1-p)^{i-1}$, $i = 1, 2, \cdots$. Explain a procedure to simulate X. (05)

Q.4 (a) Suppose lifetime random variable T follows gamma distribution with scale parameter λ and shape parameter α. Show that hazard function of T is an increasing function of t when $\alpha > 1$ and decreasing function of t when $0 < \alpha < 1$ and constant when $\alpha = 1$. (05)

(b) Define DFR class of distributions. Show that, distribution function of a lifetime random variable belongs to DFR class of distributions if $f'(t) > -\frac{f^2(t)}{\bar{F}(t)}$ and hence or otherwise show that a lifetime random variable having density function (04)

$$f(t) = \begin{cases} \frac{\beta\alpha^{\beta}}{t^{\beta+1}} & , \ t > 0, \ \alpha > 0, \ \beta > 0 \\ 0 & , \ otherwise \end{cases}$$

belongs to DFR class of distributions.

(c) Define IFR class of distributions. Check whether a lifetime random variable having survival function, (06)

$$\bar{F}(t) = \begin{cases} e^{-\lambda t} & ; \ t < t_0 \\ e^{-(3\lambda t - 2\lambda t_0)} & ; \ t_0 \le t \le 2t_0 \\ e^{-2\lambda t} & ; \ t > 2t_0 \end{cases}$$

belongs to IFR class of distributions.

Q.5 (a) Let T be a lifetime random variable with survival function $\bar{F}(t)$. Show that if $\bar{F}(t)$ belongs to IFR class of distributions then $\left[\bar{F}(t)\right]^{1/t}$ is decreasing function of t. (05)

(b) Let $T_1, T_2, \cdots, T_n$ be the independent and identically distributed random variables with Distribution $U(0, \theta)$. Define $T_{(1)}$ as a first order statistic. Obtain the survival function of $T_{(1)}$ and hazard function of $T_{(1)}$. Determine the nature of hazard function . (05)

(c) If lifetime random variable T follows Weibull distribution with shape parameter β and scale parameter α. Show that, $Y = log[-ln(\bar{F}(t))]$ linearizes the survival function $\bar{F}(t)$ and hence obtain estimates of parameters β and α. (05)

Q.6 (a) For two state vectors $\underline{x} = (x_1, x_2, \cdots, x_n)$ and $\underline{y} = (y_1, y_2, \cdots, y_n)$ of a coherent system with structure function $\phi(\underline{x})$ show that, (08)

i. $\phi\left(Max(\underline{x}, \underline{y})\right) \ \ge \ Max(\phi(\underline{x}), \phi(\underline{y}))$

ii. $\phi\left(Min(\underline{x}, \underline{y})\right) \ \le \ Min(\phi(\underline{x}), \phi(\underline{y}))$

Also identify and provide proper reasons, whether equality holds in (i) and (ii) for any of the coherent system.

(b) For a coherent system of five components, the minimal path vectors are given by, (1,0,0,1,0), (0,1,0,0,1), (1,0,1,0,1), (0,1,1,1,0). Obtain the cut vectors and minimal cut vectors for the system. (07)

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