Q. P. Code: 50442

Time: 3 Hours] **[Total Marks:80**

Instructions:

(1) Attempt any two questions from each section.

(2) All questions carry equal marks. Scientific calculator can be used.

(3) Answer to Section-I and Section-II should be written in the same answer book

Section-I

Que. 1 (a) Define: Absolute error, Relative error and Percentage error.
Use the series $\log_e\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right)$ to compute the value of $\log(1.2)$ correct to seven decimal places and find the number of terms retained.

(b) Convert the octal number $(5701.46)_8$ to the binary form and then convert to the hexadecimal form.

Que. 2 (a) Describe the Birge-Vieta method to determine a real number p such that $(x-p)$ is a factor of the polynomial equation $P_n(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$, where $a_0 \neq 0$ and $a_0, a_1, a_2, ..., a_n$ are real numbers.

(b) Perform two iterations of the Newton-Raphson method to solve the following system of non-linear equations:
$4x^2 + 2xy + y^2 = 30$ and $2x^2 + 3xy + y^2 = 3$. Use initial approximation $x_0 = -3$ and $y_0 = 2$.

Que. 3 (a) Describe the Power method to determine the largest eigen value in magnitude of the square matrix $A = [a_{ij}]$ of order n.

(b) Use Crout's method to solve the following system of linear equations.

$$\begin{aligned} 10x + 3y + 4z &= 15 \\ 2x - 10y + 3z &= 37 \\ 3x + 2y - 10z &= -10. \end{aligned}$$

Que. 4 (a) Determine the step size h that can be used in the tabulation of a function $f(x)$, $a \le x \le b$, at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than ϵ.

(b) Use Newton's divided difference formula to find the fourth degree curve passing through the points $(-4, 1245)$, $(-1, 33)$, $(0, 5)$, $(2, 9)$ and $(5, 1335)$.

Section-II

Que. 5 (a) Derive Newton-Cotes quadrature formula and use it to derive Trapezoidal rule for numerical integration.

(b) Derive two-point Gaussian quadrature formula to evaluate the integral $\int_{-1}^{1} f(x)dx$.

Use this formula to evaluate $\int_0^2 \frac{1}{x^3 + 2x + 5}\,dx$.

[Turn over

Mathematics

Extra·

Que. 6 (a) Using the least-squares method, obtain the normal equations to find the values of a, b, c and d when the curve $y = d + cx^3 + bx^6 + ax^9$ is to be fitted for the data points $(x_i, y_i), i = 1, 2, 3......., n$.

(b) Obtain the least squares quadratic approximation to the function $y(x) = \sin x$ on $[0, \pi/2]$ with respect to the weight function $W(x) = 1$.

Que. 7 (a) Derive the Adams-Bashforth predictor formula to solve the differential equation $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(b) Given $\frac{dy}{dx} = x^2 + y^2 - 2$ with $y(-0.1) = 1.0900$, $y(0) = 1.0000$, $y(0.1) = 0.8900$ and $y(0.2) = 0.7605$. Compute $y(0.3)$ correct upto four decimal places using Milne's predictor-corrector method.

Que. 8 (a) Derive a numerical method (Crank-Nicolson's method) to obtain the numerical solution of one dimensional heat equation with initial and boundary conditions.

(b) The Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ satisfies the conditions $u(x, 0) = 0$, $u(x, 4) = 8 + 2x$, $u(0, y) = \frac{1}{2}y^2$ and $u(4, y) = y^2$. Using Liebmann's method find the values of $u(i, j)$, $i = 1, 2, 3$; $j = 1, 2, 3$, correct to two places of decimals.