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Estimation of P (X ≤ Y) for the Uniform Distribution in the Presence of Outliers

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ABSTRACT The maximum likelihood and the uniformly minimum variance unbiased estimator (UMVUE) of P (X ≤ Y) are derived, where both X and Y have uniform distribution and outliers are generated from Generalized Uniform Distribution (GUD). It is shown that UMVUE is better than MLE when one parameter of GUD is known. When both parameters of the GUD are unknown, P (X ≤ Y) is estimated by using mixture estimate. It is shown that estimator of P (X ≤ Y) is consistent.

Keywords Outliers; GUD; UMVUE; MLE; MSE.

1. Introduction

The estimation of R = P (X ≤ Y) plays an important role in reliability analysis. For example, when X is stress (or demand) and Y is strength (or supply), R is taken as the measure of performance of the system.

Ooms and Moore [2] had shown that as a plant develops into the reproductive phase of growth, a mat of smaller roots grows near the surface to a depth of (1/6)th maximum depth achieved. Normally the mass of a root has a uniform distribution. Dixit et al. [1] assumes that a set of random variables(X_1, X_2, ..., X_n) represents the masses of roots, where some of these roots have different masses. Therefore, those masses have different uniform distributions with unknown parameters. Hence, we can assume that the some (say k) different observations out of n random variables are present and that these k observations are distributed with p.d.f. $f(x, \theta, \alpha)$ where

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$$g(x, \theta 1,\alpha) = \begin{cases} \dfrac{(\alpha+1)\,X^{\alpha}}{\theta_1^{(\alpha+1)}} & 0 \le x \le \theta_1\ ;\ \alpha > -1, \\ 0, & \text{Otherwise} \end{cases} \qquad (1.1)$$

and the remaining (n -k) random variables are distributed with p.d.f. $f(x,\theta_1)$

$$f(x, \theta_1) = \begin{cases} \dfrac{1}{\theta_1} & 0 \le x \le \theta_1\ ;\ \theta_1 > -1, \\ 0, & \text{Otherwise} \end{cases} \qquad (1.2)$$

Then, from (1.1) and (1.2) the joint distribution of $(X_1, X_2, \ldots, X_n)$ as in Dixit et. al. [1] is given by

$$f(x, \theta_1,\alpha) = \begin{cases} \dfrac{(a+1)^k\ \Pi_{i=1}^{n}\ I(\theta_1 - x_i)}{\theta_1^{ka+n}}\ G(\underline{x}, a)\ C(n,k) & 0 \le x \le \theta_1\ ;\ \theta_1 > -1, \\ 0, & \text{Otherwise} \end{cases} \qquad (1.3)$$

where $X = (X_1, X_2, \ldots, X_n)$, $C(n, k) = n!/((n-k)!\,k!)$, I is an indicator function, and

$$G(x, \alpha,\ \theta) = \sum_{A_1=1}^{n-k+1}\ \sum_{A_2=A_1+1}^{n-k+2} \cdots \prod_{A_1=A_{k-1}+1}^{n}\ \sum_{i=1}^{k} x_{A_i}^{\ a}$$

Let the set of random variables represents the mass of roots. We assume that these random variables are distributed with p.d.f. $h(y,\ \theta_2)$ given by

$$h(y, \theta_2) = \begin{cases} \dfrac{1}{\theta_2} & 0 \le x \le \theta_2\ ;\ \theta_2 > -1, \\ 0, & \text{Otherwise} \end{cases} \qquad (1.4)$$

Hence, from (1.4) the joint distribution of $(Y_1, Y_2, \ldots, Y_n)$ is

$$h(y, \theta_2) = \frac{\Pi_{i=1}^{n}\ I(\theta_2 - y_i)}{\theta_2^{n}} \qquad (1.5)$$

where $y = (y_1, y_2, \ldots, y_n)$. The marginal p.d.f. of X from (1.3) is given as

$$q\,(x, \theta_1, \alpha) = \begin{cases} \dfrac{b}{\theta_1} + \dfrac{b(\alpha+1)\,X^{(\alpha+1)}}{\theta_1^{(a+1)}} & 0 \le x \le \theta_1\ ;\ \theta_1 > -1, \\ 0, & \text{Otherwise} \end{cases} \qquad (1.6)$$

where $b = \frac{k}{n}$ and $\bar{b} = 1 - b$. Let $R = P[X \leq Y]$,

$$R = \int_0^{\theta_2}\int_0^{y}\left\{\frac{b}{\theta_1} + \frac{b(\alpha+1)\,X^{(\alpha+1)}}{\theta_1^{(a+1)}}\right\}\frac{1}{\theta_2}\,dxdy \quad (1.7)$$

$$= \frac{\bar{b}}{2}\frac{\theta_1}{\theta_1} + \frac{b}{(\alpha+2}\left(\frac{\theta_2}{\theta_1}\right)^{\alpha+1} \quad (1.8)$$

and our main objective is to find an estimator for P [X   Y]. An example abut dissolved oxygen (DO) in lake water of Thane city has been presented to demostrate the utility of our result.

2. *Estimation of R When a Is Known*

Theorem 2.1 *The MLE of R is given by*

$$\hat{R}_M = \frac{\bar{b}}{2}\frac{Y_n}{X_n} + \frac{b}{(\alpha+2}\left(\frac{Y_n}{X_n}\right)^{\alpha+1} \quad (2.1)$$

Proof. If a is known, then the maximum likelihood estimator of θ_1 is $X_{(n)} = Max\,(X_1, X_2, \ldots, X_n)$; See Dixit et al. [1]. From (1.5) the MLE of θ_2 is $Y_{(n)} = Max\,(Y_1, Y_2, \ldots, Y_n)$. Hence the MLE of R is given by

$$R_M = \frac{\bar{b}}{2}\frac{Y_n}{X_n} + \frac{b}{(\alpha+2}\left(\frac{Y_n}{X_n}\right)^{\alpha+1} \quad (2.2)$$

Theorem 2.2 *$\hat{R}_U$ is the UMVUE of R given by*

$$\hat{R}_M = \frac{\bar{b}}{2}\frac{\theta_2^*}{\theta_1^*} + \frac{b}{(\alpha+2}\left(\frac{\theta_2^*}{\theta_1^*}\right)^{\alpha+1} \quad (2.3)$$

where the UMVUE of θ_1 and θ_2 given by

$$\theta_1^* = \frac{e+2}{e-1}\,X_{(n)} \quad \textit{and} \quad \theta_2^* = \frac{e+2}{e-1}\,Y_{(n)}$$

respectively.

Proof. Let $T = \frac{Y(n)}{X(n)}$ and the distribution of T is given by

$$h_1(t) = \begin{cases} \left(\frac{\theta_1}{\theta_2}\right)^n \frac{e+1}{e+n+1} t^{n-1}, & 0 \le t < \frac{\theta_2}{\theta_1} \\ \left(\frac{\theta_2}{\theta_1}\right)^{e+1} \frac{e+1}{e+n+1} t^{-e-1}, & \frac{\theta_2}{\theta_1} \le t < \infty \end{cases} \tag{2.4}$$

where $e = \alpha k + n - 1$. Hence,

$$E(T^r) = \left(\frac{\theta_2}{\theta_1}\right)^r \frac{n(e+1)}{(n+r)(e-r+1)} \tag{2.5}$$

From (1.1) and (1.5), one can see easily that for known a, X(n) is sufficient and complete for θ_1, and Y(n) is sufficient and complete for θ_2. Hence, by using the Lehmann-Sheffe's theorem, we can obtain the UMVUE of R in the presence of outliers if a is known. The UMVUE of R is given by

$$R_U = \frac{\bar{b}}{2}\frac{\theta_2^*}{\theta_1^*} + \frac{b}{(\alpha+2)}\left(\frac{\theta_2^*}{\theta_1^*}\right)^{\alpha+1} \tag{2.6}$$

where

$$\theta_1^* = \frac{e+2}{e-1} X_{(n)} \qquad and \qquad \theta_2^* = \frac{e+2}{e-1} Y_{(n)}$$

Hence,

$$\hat{R}_U = \frac{e(n+1)}{n(e+1)} \frac{b}{2} \frac{Y_n}{X_n} + \frac{(e-\alpha)(n+\alpha+1)}{n(e+1)} \frac{b}{\alpha+2} \left(\frac{Y_n}{X_n}\right)^{\alpha+1} \tag{2.7}$$

is the UMVUE of R, and

$$E(\hat{R}_M) = \frac{\bar{b}}{2}\frac{\theta_2}{\theta_1} + \frac{b}{\alpha+2}\left(\frac{\theta_2}{\theta_1}\right)^{\alpha+1} = R \tag{2.8}$$

Theorem 2.3 RU is consistent estimator of R.

Proof.

$$V(\hat{R}_U) = E(R_U - R)^2 = \frac{\bar{b}^2}{2}\left(\frac{\theta_2}{\theta_1}\right)^2 \left[\frac{e^2(n+1)^2}{n(e+1)(n+2)(e-1)} - 1\right] +$$

$$\left(\frac{b}{\alpha+2}\right)^2 \left(\frac{\theta_2}{\theta_1}\right)^{2\alpha+2} \left[\frac{(e-\alpha)^2(n+\alpha+1)^2}{n(e+1)(n+2\alpha+2)(e-\alpha-1)} - 1\right] +$$

$$\frac{b\bar{b}}{\alpha+2}\left(\frac{\theta_2}{\theta_1}\right)^{\alpha+2} \left[\frac{e(e-\alpha)(n+1)(n+\alpha+1)}{n(e+1)(n+\alpha+2)(e-\alpha-1)} - 1\right] + \tag{2.9}$$

Thus As'$n \to \infty$ and $k \to \infty$, $b \to \gamma$ and $\bar{b} \to \bar{\gamma}$, where $0 < \gamma < 1$, $\bar{\gamma} = 1 - \gamma$, Since

$$\frac{e^2 (n+1)^2}{n(e+1)(n+2)(e-1)} = \frac{(\alpha k + n - 1)^2 (n+1)^2}{n(\alpha k + n)(n+2)(\alpha k + n - 2)}$$

$$= \left[(a\bar{b} + 1 - \frac{1}{n})^2 (1 + \frac{1}{n})^2 \right] / [(a\bar{b} + 1)(1 + \frac{1}{n})(a\bar{b} + 1 \frac{1}{n})] \quad (2.9)$$

we have

$$\lim_{k \to \infty n \to \infty} \frac{e^2 (n+1)^2}{n(e+1)(n+2)(e-1)} = 1$$

Similarly, as $k \to \infty$ and $n \to \infty$

$$\frac{(e-\alpha)^2 (n+\alpha+1)^2}{n(e+1)(n+2\alpha+2)(e-2\alpha-1)} \to 1 \quad \frac{e(e-\alpha)(n+1)(n+\alpha+1)^2}{n(e+1)(n+\alpha+2)(e-\alpha-1)} \to 1$$

Hence, $V(\hat{R}_U) \to 0$ as $k \to \infty$ and $n \to \infty$, i.e., $\hat{R}_U$ is consistent estimator of R.

Theorem 2.4 $\hat{R}_M$ is consistent estimator of R.

Proof. From (2.1) and (2.4)

$$E(\hat{R}_M) = \frac{\bar{b}}{2} E(t) \frac{b}{\alpha+2} E(t^{\alpha+1}) = \frac{n(e+1)}{e(n+1)} \frac{\bar{b}}{2} \frac{\theta_2}{\theta_1}$$

$$+ \frac{n(e+1)}{(e-\alpha)(n+\alpha+1)} \frac{b}{\alpha+2} \left(\frac{\theta_2}{\theta_1}\right)^{\alpha+1}$$

$$\to \frac{\bar{\gamma}}{2} \frac{\theta_2}{\theta_1} + \frac{\gamma}{\alpha+2} \left(\frac{\theta_2}{\theta_1}\right)^{\alpha+1}$$

thus, $E(\hat{R}_M) \to R$. Further consider MSE $(R_M) = E(\hat{R}_M - R)^2 = E(R_M)^2 - 2RE(\hat{R}_M) + R^2$. Now,

$$E(\hat{R}_M) = \frac{\bar{b}^2}{4} Et^2 \frac{b^2}{(\alpha+2)^2} Et^{2\alpha+2} + \frac{\bar{b}b}{\alpha+2} Et^{\alpha+2} = \frac{\bar{b}}{4} \left(\frac{\theta_2}{\theta_1}\right)^2 \left[\frac{n(e+1)}{(e-1)(n+2)}\right]$$

$$+ \frac{b}{\alpha+2} \left(\frac{\theta_2}{\theta_1}\right)^{2\alpha+2} \left[\frac{(e+1)n}{(n+2\alpha+2)(e-2\alpha-1)}\right]$$

$$+ \frac{\bar{b}b}{\alpha+2} \left(\frac{\theta_2}{\theta_1}\right)^{\alpha+2} \left[\frac{(e+1)n}{(n+\alpha+2)(e-\alpha-1)}\right] \quad (2.10)$$

$$E(\hat{R}_M) = \left[\frac{n(e+1)}{e(n+1)} \frac{\bar{b}}{4} \frac{\theta_2}{\theta_1} + \frac{n(e+1)}{(e-a)(n+\alpha+1)} \frac{b}{a+2} \left(\frac{\theta_2}{\theta_1}\right)^{\alpha+1}\right] \quad (2.11)$$

$$R^2 = \left[\frac{b}{2}\frac{\theta_2}{\theta_1} + \frac{b}{\alpha+2}\left(\frac{\theta_2}{\theta_1}\right)^{\alpha+1}\right]^2$$

$$= \frac{b^2}{2}\left(\frac{\theta_2}{\theta_1}\right)^2 + \left(\frac{b}{\alpha+2}\right)^2 \frac{\theta_2}{\theta_1} + \frac{\bar{b}b}{\alpha+2}\left(\frac{\theta_2}{\theta_1}\right)^{\alpha+2} \tag{2.12}$$

From (2, 10), (2.11) and (2.12) the MSE ($\hat{R}_M$) is

$$\frac{\bar{b}^2}{2}\left(\frac{\theta_2}{\theta_1}\right)^2\left[\frac{n(e+1)}{(e-1)(n+2)} - 2\,\frac{n(e+1)}{(e-1)(n+2)} + 1\right] + \left(\frac{b}{\alpha+2}\right)^2\left(\frac{\theta_2}{\theta_1}\right)^{2\alpha+2} \quad \frac{\theta_2}{\theta_1}$$

$$\left[\frac{n(e+1)n}{(n+2\alpha+2)(e-2\alpha-1)} - 2\,\frac{n(e+1)}{(e-\alpha)(n+\alpha+1)} + 1\right] + \frac{\bar{b}b}{\alpha+2}\left(\frac{\theta_2}{\theta_1}\right)^{\alpha+2}$$

$$\left[\frac{n(e+1)n}{(n+\alpha+2)(e-\alpha-1)} - \frac{n(e+1)}{(e-\alpha)(n+\alpha+1)} - \frac{n(e+1)}{e(n+1)} + 1\right]$$

As $k \to \infty$ and $n \to \infty$, MSE ($\hat{R}_M$) 0, i.e., $\hat{R}_M$ is consistent estimator of R.

Corollary 2.1 $\hat{R}_U$ is more efficient than $\hat{R}_M$.

Proof.

$$E(\hat{R}_M - R)^2 = E(\hat{R}_M - R + \hat{R}_U - \hat{R}_U)^2 = E[(\hat{R}_U - R) + (\hat{R}_M - \hat{R}_U)]^2$$

$$= E(\hat{R}_U - R)^2 + E(\hat{R}_M - \hat{R}_U)^2$$

$$= V(\hat{R}_U) + E(\hat{R}_M - \hat{R}_U)^2 \tag{2.16}$$

Thus MSE ($\hat{R}_M$) $\geq$ ($\hat{R}_U$), i.e. $\hat{R}_U$ is more efficient than MLE $\hat{R}_M$.

3. Estimation of R When a Is Unknown

Dixit et al. [1] had given the mixture estimator of θ_1 and α as follows :

$$\tilde{\theta}_1 = x_{(n)} \text{ and } \tilde{\alpha} = \frac{4m_1 - 2x_{(n)}}{2bx_{(n)} - 2m_1 + bx_{(n)}} \quad ; \text{ where } m_1 = \frac{\sum_{i=1}^{n} x_i}{n}.$$

Further, they have shown that $\tilde{\theta}_1$ and $\tilde{\alpha}$ are consistent estimators and also E $\tilde{\alpha} = \alpha$ as $k \to \infty$ and $n \to \infty$. In this case, the mixture estimator of R is

$$\tilde{R} = \frac{b}{2}\frac{y_n}{x_n} + \frac{b}{\tilde{\alpha}+2}\left(\frac{y_n}{x_n}\right)^{\tilde{\alpha}+1} \tag{3.1}$$

Theorem 3.1 $\tilde{\tilde{R}}$ is a consistent estimator.

Proof . Let $\tilde{R} = f\, X_{(n)}, Y_{(n)}$. Suppose $EX_{(n)} = \lambda_1$ and $EY_{(n)} = \lambda_2$. Expand the function $f X_{(n)}$, $Y_{(n)}$ around (λ_1, λ_2) by Taylor's series then

$$\tilde{\tilde{R}} = f(\lambda_1, \lambda_2) + (X_{(n)} - \lambda_1)\left(\frac{\partial f}{\partial X_{(n)}}\right)_{X_{(n)}=\lambda_1,\, Y_{(n)}=\lambda_1} + (Y_{(n)} - \lambda_1)\left(\frac{\partial f}{\partial Y_{(n)}}\right)_{X_{(n)}=\lambda_1,\, X_{(n)}=\lambda_1} + 0\left(\frac{1}{n}\right) \quad (3.2)$$

From (3.2), it can be easily seen that $E\tilde{\tilde{R}} \to R$ as $k \to \infty$ and $n \to \infty$ Next,

$$E(\tilde{\tilde{R}} - R)^2 = V(X_{(n)})\left(\frac{\partial f}{\partial Y_{(n)}}\right)^2_{X_{(n)}=\lambda_1,\, Y_{(n)}=\lambda_1} + V(Y_{(n)})\left(\frac{\partial f}{\partial Y_{(n)}}\right)^2_{X_{(n)}=\lambda_1,\, Y_{(n)}=\lambda_1} + 2Cov(X_{(n)}, Y_{(n)})\frac{\partial f}{\partial X_{(n)}}\frac{\partial f}{\partial Y_{(n)}} + 0\left(\frac{1}{n}\right) \quad (3.2)$$

Since $X_{(n)}$ and $Y_{(n)}$ are independent, $Cov(X_{(n)}, Y_{(n)}) = 0$

$$\frac{\partial f}{\partial Y_{(n)}} = -\frac{\bar{b}}{2}\frac{\lambda_2}{\lambda_1^2} + \frac{b}{a+2}\left(\frac{\lambda_2}{\lambda_1}\right)^{\alpha+1}\left(\frac{\lambda_2}{\lambda_2^2}\right); \qquad \lambda_2 = \frac{n\theta_2}{n+1}, \quad \lambda_1 = \frac{(e+1)\theta_2}{e+2}$$

$$= -\frac{\bar{b}}{2}\left(\frac{n\theta_2}{n+1}\right)\frac{(\alpha k+n+1)^2}{\theta_1(\alpha k+n)^2} - \frac{b(\alpha+1)(\alpha k+n+1)^2}{\alpha+2\,\theta_1^2(\alpha k+n)^2} \times \left[\frac{n\theta_2}{n+1}\,\frac{(\alpha k+n+1)}{\theta_1(\alpha k+n)}\right]^a$$

$$= -\frac{\bar{b}}{2}\left(\frac{n(\alpha k+n+1)^2}{(\alpha k+n)^2(n+1)}\right)\frac{\theta^2}{\theta_1^2} - \frac{b(\alpha+1)(\alpha k+n+1)^2}{\alpha+2\,\theta_1^2(\alpha k+n)^2} \times \left[\frac{n\theta_2}{n+1}\,\frac{(\alpha k+n+1)}{\theta_1(\alpha k+n)}\right]^\alpha\left(\frac{\theta^2}{\theta_1}\right)^\alpha \quad (3.4)$$

$$\frac{\partial f}{\partial Y_{(n)}} = -\frac{b}{2\lambda_1} + \frac{b(\alpha+1}{\alpha+2}\left(\frac{\lambda_2}{\lambda_1}\right)^\alpha\left(\frac{1}{\lambda_1}\right); \qquad \lambda_2 = \frac{n\theta_2}{n+1}, \quad \lambda_1 = \frac{(e+1)\theta_1}{e+2}$$

$$= -\frac{\bar{b}}{2}\,\frac{(\alpha k+n+1)}{\theta_1(\alpha k+n)} + \frac{b(\alpha+1)}{\alpha+2} \times \frac{(\alpha k+n+1)}{(\alpha k+n)\theta_1}\left[\frac{n\theta_2}{n+1}\,\frac{(\alpha k+n+1)}{\theta_1(\alpha k+n)}\right]^\alpha$$

$$= \frac{\bar{b}}{2\theta_1}\,\frac{(\alpha k+n+1)}{\theta_1(\alpha k+n)} + \frac{b(\alpha+1)}{\alpha+2}\,\frac{(\alpha k+n+1)}{(\alpha k+n)}\left[\frac{n}{n+1}\,\frac{(\alpha k+n+1)}{(\alpha k+n)}\right]^\alpha\left(\frac{\theta^2}{\theta_1}\right)^\alpha \quad (3.5)$$

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$$V(X_{(n)}) = \frac{(e+1)\,\theta_1^2}{(e+3)\,(e+2)^2} \quad \text{and} \quad V(Y_{(n)}) = \frac{n\theta_2^2}{(n+1)\,(n+2)^2} \tag{3.6}$$

As $k \to \infty$ and $n \to \infty$, $\frac{\partial f}{\partial (X_{(n)})} \to c_1$, $\frac{\partial f}{\partial (X_{(n)})} \to c_2$, $V(X_{(n)}) \to 0$ and $V(Y_{(n)}) \to 0$. Hence, $\tilde{R}$ is consistent estimator.

Note : If k is unknown, then it can be selected by evaluating the likelihood function for each k, and choosing the one that maximizes the likelihood.

4. An Example

In a clean environment the dissolved oxygen (DO) mg/litre of lake water in Thane city has been studied. It is found to be (as (1.4) uniformly distributed over $(0, \theta_2)$. However, the study of (DO) in a polluted environment is observed with changed behavior, and it is found to fol low (as (1.3)) GUD with k outliers, a = 1.2 and q1. The shape parameter a is found to be inversely related to the pollution level in the water.

Sample from (1.4)
2.03, 0.73, 1.57, 1.27, 1.30, 1.83, 1.57, 2.03, 3.0, 1.30, 1.87, 0.93.

Sample from (1.3)
1.33, 1.13, 0.00, 0.05, 3.72, 4.81, 2.77, 2.04, 4.85, 1.94, 0.0, 1.28.

For α unknown, we obtain

k	$\hat{\alpha}$	$\hat{\theta}_1$	$L(x,\theta_1)$
1	1.39273	3.22427	.00010095
2	1.52460	3.19934	.00041255
3	1.63889	3.17734	.00113503
4	1.73889	3.15827	.00181978
5	1.82712	3.14194	.00167344

Thus the likelihood function as in (1.3) is maximized for k = 4, and we have following if a is unknown :

$\hat{\alpha} = 1.73889$, $\hat{\theta} = 3.15827$, $\hat{R}_U = 0.87118$, $\hat{R}_M = 0.902165$

For a known, we obtain

k	L ($\underline{x}$,θ_1)	$\hat{\theta}_1$
1	.0001509	3.222727
2	.0008333	3.208333
3	.0003179	3.192308
4	.00033178	3.17897
5	.0002100	3.16667

Thus the likelihood function as in (1.3) is maximized for k = 2, and we have the following if α= 1.2 :

$$\alpha = 1.2, \quad \theta = 3.20833, \quad R_U = 0.8293026, \quad R'_M = 0.8234636$$

Hence an application of our result would yield an extremely useful suggestion for the authorities to decide when the lake is to be cleaned. The higher the value of the probability is, the higher the pollution level will be. Especially if bio-remedial cleaning is to be performed, it would be one of the most time, money and labor saving gadgets.

5. Conclusion

Here we conclude that if α is known then MSE of R'_U is less than MSE of $\hat{R}_M$. Hence the UMVUE of R should be selected. If α is unknown then we can select the mixture estimator of R because if is consistent and easy to calculate.

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