Graph Theory QP Code : AU-6400

(2½ Hours) [Total Marks : 60

N.B. : (1) **All** questions are **compulsory.**
(2) **Figures** to the **right** indicate **marks** for respective **subquestions.**

1. (a) If G is a self complementary graph of order p, show that G is connected and $p \equiv 0 \; or \; 1 (mod \; 4)$ (7)
   (b) Attempt any **two** questions:
      (i) Show that every nontrivial graph contains at least two vertices which are not cut vertices. (4)
      (ii) Prove that an edge e of a graph G is a cut edge of G if and only if e is acyclic and hence prove that every edge in a tree is a cut edge. (4)
      (iii) Prove that every (p, q) graph with $q \geq p$ contains a cycle. (4)
      (iv) If G is a simple graph on n vertices and $\delta(G) \geq \frac{n-1}{2}$ then prove that G is a connected graph where $\delta(G)$ denotes the minimum degree of G. Give an example of a graph with $\delta(G) \geq \frac{n-2}{2}$ which is not connected. (4)

2. (a) Let $\tau(G)$ denote the number of spanning trees of a graph G. If $e \in E(G)$ is not a loop then prove that $\tau(G) = \tau(G - e) + \tau(G.e)$. (7)
   (b) Attempt any **two** questions:
      (i) Show that if a tree has exactly 2 pendant vertices then, all the other vertices have degree 2. (4)
      (ii) If T is a tree with n vertices whose degree sequences is $d_1, d_2, \cdots d_n$ then prove that $\sum_{i=1}^{n} d_i = 2(n-1)$. (4)
      (iii) If T is a spanning tree of a connected graph G and e is an edge of G that is not in T, then prove that $T + e$ contains a unique cycle that contains the edge e. (4)
      (iv) State Cayley's formula for Spanning trees. Using the formula or otherwise, find the number of spanning trees of a graph of order 4 obtained by joining a new vertex to any arbitrary vertex of K_3. Justify your answer. (4)

3. (a) If G is a graph on $p \geq 3$ vertices with $\delta(G) \geq \frac{p}{2}$ where $\delta(G)$ denotes the minimum degree of G then show that G contains a Hamiltonian cycle. (7)
   (b) Attempt any **two** questions:
      (i) Show that $K_{m,n}$ is not Hamiltonian if and only if $m \neq n$. (4)
      (ii) Show that if the closure of a graph G is complete then G is Hamiltonian. (4)
      (iii) Show that the graph associated with the legal moves of a knight on a $m \times n$ chess board is a bipartite graph and hence show that the graph of legal moves of a 5×7 chess board is not Hamiltonian. (4)
      (iv) Show that in a complete graph K_{2n+1}, there are n edge disjoint Hamiltonian cycles. (4)

4. Attempt any **three** questions:
   (a) Show that a nontrivial graph is bipartite if and only if it contains no odd cycle. (5)
   (b) If G is a Hamiltonian graph then for every non empty proper subset S of $V(G)$, prove that $\omega(G - S) \leq |S|$. (5)
   (c) Prove that the Cube graph Q_n has 2^n vertices and $n.2^{n-1}$ edges for $n \geq 1$. (5)
   (d) For any simple graph G, prove that $\kappa(G) \leq \kappa'(G) \leq \delta(G)$ where $\kappa(G)$ denotes the vertex connectivity of G and $\kappa'(G)$ denotes edge connectivity of G and $\delta(G)$ denotes the minimum degree of G. (5)

Con. 10020(b)-14.