B.N.Bandodkar College of Science,Thane

Second Semester End Examination March 2013

USMT 202

Duration: 3 hours **Max.Marks: 60**

Instructions to the candidates:

1. **All questions are compulsory.**
2. **Figures to the right indicate full marks.**

Q.1 Attempt any three of the following

(i) Prove that the relation of ' congruent modulo n ' for any fixed positive integer n, is an equivalence relation on Z. (5)

(ii) Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$, $n \geq 3$ with $a_1 = 1$ and $a_2 = 3$. (5)

(iii) Write the characteristic equation for $a_n = 2a_{n-1} - a_{n-2}$, $n \geq 3$, $a_1 = 0$ and $a_2 = 2$. Also find a_3 and a_4 . (5)

(iv) Prove that any two equivalence classes of X are either disjoint or identical. (5)

(v) Define Stirling number of second kind . Find S(4,1) , S(3,1), S(3,2), S(3,3). (5)

(vi) Prove that $S(n,2) = 2^{n-1} - 1$, $n \geq 2$. (5)

Q.2 Attempt any three of the following

(i) Define multinomial number. How many different arrangements can be made by using all the letters of the word EXAMINATIONS. (5)

(ii) Define permutation. Write the permutation $\sigma = (2\ 4)(1\ 3\ 2)(4\ 5)(3\ 4)$ of S_5, in the standard form. (5)

(iii) Define transposition of a permutation. Express $\sigma = (3\ 5\ 7)(2\ 3\ 1\ 6\ 5\ 4)(6\ 8\ 3\ 2\ 1)$ in terms of product of transpositions. Hence, find $sgn(\sigma)$. (5)

(iv) Find the total number of derangements on 4 symbols. (5)

(v) Find the number of integers between 1 and 250 , that are not divisible by 2, 5 and 7. (5)

(vi) Define multinomial number. Calculate the coefficient of $x^3y^4z^3w$ in the expansion of $(x + y + z + w)^{11}$. (5)

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Q.3 **Attempt any three of the following**

(i) Define an irreducible polynomial. Show that $x^2 - 9$ is reducible in $\Re[x]$. (5)
(ii) Find the greatest common divisor (g.c.d) of $f(x) = x^8 - 1$ and $g(x) = x^6 - 1$. (5)
(iii) State and prove rational root theorem. (5)
(iv) Show that $\sqrt{5}$ is an irrational number. (5)
(v) State and prove remainder theorem (5)
(vi) Prove that complex roots occurs in conjugate pairs. (5)

Q.4 **Attempt any three of the following**

(i) Find the inverse of $\sigma = (3\ 4\ 2)(1\ 6\ 5)$ and $\tau = (1\ 3\ 2)(4\ 6\ 5)$, in S_6. (5)
(ii) State and prove principle of inclusion and exclusion. (5)
(iii) Let X be a finite set and f: $X \rightarrow X$ is given by $f^2(x) = x$, for $x \in X$. Show that f is a bijection. (5)
(iv) Find modulus and amplitude of $z = 4 + 3i$. Also give its polar representation (5)
(v) Prove that a polynomial of degree n has atmost n roots. (5)
(vi) In a club with 54 members, 34 members play tennis, 22 play football and 11 play cricket. Of these, 10 play both tennis and football, 6 play tennis and cricket and 4 play football and cricket. If 2 members play all the games, find how many members play none of the games. (5)

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