Duration : 2 hours **Total Marks : 60**

N.B. (1) All questions are compulsory.

Q1. A) Attempt any ONE of the following. **(08 M)**

a) Define the row echelon form of a matrix. Prove that a square matrix is invertible if & only if its row echelon form is invertible.

b) Prove that a homogeneous system of m equations in n unknowns with $m < n$ can not nave a unique solution.

B) Attempt any TWO of the following. **(08 M)**

a) Use the method of adjoint to find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 8 \end{bmatrix}.$$

b) Solve using Gauss elimination :

$x + y + z = 1,\ \ y - 2z = 0,\ x + 2y - 4z = 0.$

c) Explain graphically why the systems (i) $x + y = 1,\ x - y = 0$ & (ii) $x + y = 1,\ x + y = 2$ are not consistent.

d) Define an elementary matrix. Express $A = \begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix}$ as a product of elementary matrices.

Q.2. A) Attempt any ONE of the following. **(08 M)**

a) State & Prove subspace test.

b) Define linearly independent set in a vector space.Prove that a subset of a vector space containing a zero vector is not linearly independent.

Q.2. B) Attempt any TWO of the following. **(08 M)**

a) Show that a set of real valued continuous functions on IR is a vector space.

b) Can we express (1, 1, -1) as a linear combination of (1, 0, 0), (0, -1, 1) & (0, 2, -3) ? Justify.

c) Find the co-ordinate vector of (1, 2, 3) with respect to the basis $B = \{ (1, 1, 1), (0, 1, 1), (0, 0, 1) \}$ in IR^3

d) Prove that the union of two subspaces of a vector space is a subspace only if one of the subspace is contained into the other.

Q 3. A) Attempt any ONE of the following. **(08 M)**

a) Define the determinant of n × n matrix. Prove that the equation of a line passing through the points (a,b) and (c,d) is given by Det $\begin{bmatrix} x & y & 1 \\ a & b & 1 \\ c & d & 1 \end{bmatrix} = 0$

b) Prove the Rank - Nullity theorem for matrices.

Q 3. B) Attempt any TWO of the following. **(08 M)**

a) Solve the following system of the equations using LU decomposition

$x + y + 2z = 8, \; -x - 2y + 3z = 1, \; 3x - 7y + 4z = 10$

b) Verify the Rank –Nullity theorem for $\begin{bmatrix} 1 & -1 & 1 \\ 3 & 2 & -1 \\ 4 & 1 & 0 \end{bmatrix}$

c) Prove that the system **Ax** = **0** where **x** $\in \mathbb{R}^n$ and **A** is n × n.
If rank (**A**) = r < n, how many linearly independent vectors are required to generate a solution of **Ax** = **0**. Justify your answer

d) Can we find out the nature of the solution space of a non homogeneous,square system by knowing the value of determinant of its coefficient matrix? Explain.

Q 4. Attempt any THREE of the following. **(12 M)**

a) For what value of **k** in the given below system has no solution.

$x + y + z = 1, \; -x - y - z = 2, \; x + y + z = k$

b) Reduce the matrix $\begin{bmatrix} 1 & 5 & 11 \\ 2 & 10 & 22 \\ 3 & 15 & 33 \end{bmatrix}$ into its row echelon form.

c) If **W** = {(x, y, z) $\in \mathbb{R}^3$ | $x + y - z = 0$} Find a Basis & the dimension of **W**.

d) Determine whether the set **W** = { e^x, e^{2x}, e^{3x} } is linearly independent.

e) Explain why the solution set of a non homogeneous system is not a vector space.

f) Explain using Rank – Nullity theorem why the maximum possible rank of m × n matrix is the minimum of { m, n}.

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