BNBUSMT3T1/02/2022/ Revised

Duration: 2 1/2 Hours **[Total Marks: 60]**

N.B.: 1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 a) Attempt any ONE question from the following: **(08)**

i. State and prove Cauchy Criterion for convergence of series.

ii. State and prove Comparison test.

b) Attempt any TWO questions from the following: **(08)**

i. Prove that $\sum_{n=1}^{\infty}\left(\frac{1}{n^p}\right)$ converges if and only if $p > 1$.

ii. Determine the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{(-1)^n}{5^n}\right)$.

iii. Determine whether a series $\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1}}{3n^2}\right)$ is convergent.

iv. Write 0.123123........ as a series and discuss the convergence.

Q.2 a) Attempt any ONE question from the following: **(08)**

i. Let $f:[a,b] \to \mathbb{R}$ be a bounded function. If f is continuous then prove that f is R-integrable.

ii. Let $f:[a,b] \to \mathbb{R}$ be a bounded function and $a < c < b$. Prove that f is R-integrable on [a, b] if and only if f is R-integrable on [a, c] and [c, b].

b) Attempt any TWO questions from the following: **(08)**

i. If $f:[0,1] \to IR$ be a function such that $f(x) = 4x + 1$. Let P be a partition with P: 0, 0.5, 1. Find upper sum U(f, P).

ii. Let $f:[a,b] \to \mathbb{R}$ be R-Integrable function. If P and Q are partitions of [a, b] such that $P \subseteq Q$ then prove that $L(P,f) \leq L(Q,f)$.

iii. Prove that $f:[0,1] \to \mathbb{R}$, $f(x) = 3x$ is Riemann Integrable using Reimann sum.

iv. Let $f:[a,b] \to \mathbb{R}$ be a bounded function be such that f(x) = 2 for all x. Prove that f is R-Integrable.

Q.3 a) Attempt any ONE question from the following: **(08)**

i. State and prove First Fundamental theorem of calculus.

ii. State and prove Mean value theorem of integration

b) Attempt any TWO questions from the following: **(08)**

i. Examine convergence of $\int_0^{\infty} \frac{x^2}{(1+x)^3} dx$.

ii. Prove that $\lceil n = (n-1)!$, where n is a natural number.

iii. Prove that $\beta(m,1) = \frac{1}{m}$.

iv. Find area of the region bounded by $y = x^2$ and y = 4.

Q.4 Attempt any THREE questions from the following: **(12)**

a) Prove that every absolutely convergent series is convergent.

b) Discuss the convergence of $\sum_{n=1}^{\infty} \left(\frac{\log n}{2n^3}\right)$.

c) If f and g are R-integrable functions on [a, b] then prove that f - g is also R-integrable on [a, b].

d) Find the value of improper integral $\int_0^{\infty} x^9 e^{-x} dx$.

e) Find the value of the integral $\int_0^1 x^6 (1-x)^5 dx$.

f) Discuss the convergence of $\int_0^2 \frac{1}{2-x} dx$.

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