B.N. BANDODKAR COLLEGE OF SCIENCE, THANE.

DEPT. OF MATHEMATICS

TYBSC PRELIMINARY EXAMINATION 2010-11

DURATION: 3 HRS MARKS: 100

PAPER II (ALGEBRA)

N.B. All the questions are compulsory.

Q-1. Attempt any two of the following:

A] 1. Let V be the finite dimensional vector space and W be a subspace of V. Then prove that dim V/W = dim V- dim W. [7]

2. Prove that eigen values of real skew symmetric matrix are purely imaginary. [3]

B] 1. Define Normal Subgroup of a group. If H is the only subgroup of given order of finite group G, then show that H is normal subgroup of G. [6]

2. Prove that there are only two automorphism of infinite cyclic group. [4]

C] 1. Let R be a commutative ring with unity. Show that regular elements form an abelian group. [4]

2. Show that it is impossible to have an Integral Domain with six elements. [3]

3. Prove that in Euclidean Domain any two elements have a common divisor. [3]

Q-2. Attempt any two of the following:

A] 1. State and prove First Isomorphism theorem of real vector spaces. (i.e. Fundamental Theorem of homomorphism of V.S.) [7]

2.Show that characteristic value of triangular matrix are its diagonal elements. [3]

B] 1. Prove that every isometry is a composition of orthogonal transformation and a translation. [7]

2. Prove that eigen vectors associated to distinct eigen values are linearly independent. [3]

C] 1. Prove that every real symmetric matrix with distinct eigen value is orthogonally diagonalizable. [5]

2. Prove that a symmetric matrix A is positive definite if and only if all eigen values of A are positive. [5]

Q-3. Attempt any two of the following:

A] 1. Let G = < a > be a cyclic group of order n. Then prove that G= < a^m > if and only if m and n are relatively prime. [6]

2. Show that an infinite cyclic group has only two generators. [4]

B] 1. Let G be a group and H, K are subgroups of G of order p, where p is a prime number. Show that either H = k or H∩K = {e}. [3]

2. Let G be a group. Show that f: G--> G defined by f(x) = x^{-1} is an automorphism of G if and only if G is abelian. [4]

3. Let G, G' be groups and f: G--> G' a homomorphism of G onto G'. Show that f (a^{-1}) = $[f (a)]^{-1}$, for all a in G. [3]

C] 1. State and prove Lagrange's Theorem. [7]

2. Let f: (Z_m, +) --> (Z_n, +) be a homomorphism of groups and (m, n) =1 then show that f=0, where (Z_m, $+_m$), (Z_n, $+_n$) denote the groups of residue classes modulo m and n respectively. [3]

Q-4. Attempt any two of the following:

A] 1.Let G be a group and H be a subgroup of G of index 2. Show that H is a normal subgroup of G. Give an example to show that a subgroup of index 3 may not be normal sub group of G. [6]

2. Let G, G’ be groups and f: G --> G’ be a homomorphism of G onto G’. Prove that if H is a subgroup of G, then prove that f(H)={f(h): h is in H} is a subgroup of G’. If H is normal in G then show that f(H) is normal in G’. [4]

B] 1. Let G_1, G_2 be groups. Show that if G_1, G_2 are finite cyclic groups such that $(o(G_1), o(G_2)) = 1$, then show that their product $G_1 \times G_2$ is cyclic. If G_1, G_2 are finite cyclic groups such that $G_1 \times G_2$ is cyclic. Can you say that $(o(G_1), o(G_2)) = 1$? Justify your answer. [5]

2. Show that S_3 cannot be expressed as direct product of two non-trivial groups. [2]

3. If G_1 , G_2 are groups and a is in G_1 , b is in G_2 such that o(a)=m, o(b)=n then show that o((a, b)) = l.c.m.{m, n}. [3]

C] 1. State and prove Fundamental Theorem of Homomorphism of groups (OR First Isomorphism Theorem). [7]

2. If G, G’ are groups and f: G --> G’ is a homomorphism then prove that Ker f is normal subgroup of G. [3]

Q-5. Attempt any two of the following:

A] 1. Define Characteristic of a ring. Prove that Characteristic of an Integral Domain is either zero or prime. [5]

2. Prove that a finite Integral Domain is a field. [5]

B] 1. Define left and right Ideal of a ring. Prove that a field has no proper Ideal (i.e. A field has only two Ideal). [6]

2. Let R be a commutative ring and P be an Ideal of R. Prove that P is a prime Ideal of R if and only if R/P is an Intregal Domain. [4]

C] 1. Define Euclidean Domain. Prove that ring of Gaussian integers is an Euclidean Domain. [7]

2. Prove that Euclidean Domain is Unique Factorization Domain.[3]