B. N. BANDODKAR COLLEGE OF SCIENCE, THANE

T.Y.B.Sc PRELIMINARY EXAMINATION, FEB 2011

Duration: 3 hours | Total marks: 100

Note: (i) Figures on the right indicate full marks

(ii) All questions are compulsory.

(iii) Scientific calculators are allowed.

MATHS IV

1. Answer any **two** of the following questions: (20)

(a) Show that the Newton-Raphson method has second order convergence.

(b) Prove the following relations:

(i) $\Delta + \nabla = \Delta/\nabla - \nabla/\Delta$

(ii) $\Delta(f_i g_i) = f_i \Delta g_i + g_{i+1}\Delta f_i$

(c) Find the largest eigenvalue in modulus and the corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{pmatrix}$$

using the Power method.

2. Answer any **two** of the following questions: (20)

(a) If x_{k-1} and x_k are two approximations to the root of $f(x) = 0$, then show that the next approximation x_{k+1} to the root using Secant method is

$$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k) \quad for\ \ k = 1,2,\cdots$$

(b) Perform 4 iterations of the Newton-Raphson method to obtain the approximate value of $(17)^{1/3}$ starting with the initial approximation $x_0 = 2$.

(c) Perform 2 iterations of the Birge-Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$. Use the initial approximation $p_0 = 0.5$

3. Answer any **two** of the following questions: (20)

(a) Describe Cholesky method for solving numerically a system of linear equations.

(b) Transform the matrix $M = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{pmatrix}$ to tridiagonal form by Givens method.

(c) Find all the approximate values of eigenvalues of the matrix $M = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$. Obtain first 5 iterates.

4. Answer any **two** of the following questions: (20)

(a) Show that E $\equiv$ I + Δ and deduce the Gregory-Newton forward difference interpolating polynomial with usual notation

$$P_n(x) = \sum_{i=0}^{n} {}^{u}C_i \Delta^i f(x_0)$$

(b) Obtain the piecewise linear interpolating polynomial for the function f(x) defined by the data

x	1	2	4	8
$f(x)$	3	7	21	73

Hence estimate the values of f(3) and f(6).

(c) The following data represents a function f(x,y)

$y\backslash x$	0	1	3
0	−4	−3	23
2	12	13	39

Obtain the interpolating polynomial that fits the data.

5. Answer any **two** of the following questions: (20)

(a) Using Lagrange's interpolating polynomial

$$P_n(x) = \sum_{k=0}^{n} l_k(x) f(x_k)$$

where $l_k(x)$ is the Lagrange's fundamental polynomial, show that

$$P_1'(x) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

using linear interpolation.

(b) Evaluate the integral

$$I = \int_1^2 \int_1^2 \frac{dx\, dy}{x + y}$$

using the trapezoidal rule with h = k = 0.25.

(c) Find the approximate value of

$$I = \int_0^1 \frac{dx}{1 + x}$$

using (i) Trapezoidal rule and

(ii) Simpson's rule.