

B. N. BANDODKAR COLLEGE OF SCIENCE – THANE

Third Semester Theory Examination 2013

USMT 301

Duration : 2 Hrs

Marks : 60

N.B. All questions are compulsory.

Q.1

Attempt any three of the following.

- (i) If $a > 0$ and $b > 0$ then prove that $\sqrt{ab} \leq \frac{a+b}{2}$. 5
- (ii) State and prove Archimedian property in \mathbb{R} . 5
- (iii) Prove that every closed interval is a closed set. 5
- (iv) Prove that a set is closed if and only if it contains all its limit point. 5
- (v) Prove that arbitrary union of open sets is open in \mathbb{R} . 5
- (vi) Prove that the set $(0,1)$ is covered by $\{(\frac{1}{n}, 2) \mid n \in \mathbb{N}\}$. 5

Q.2

Attempt any three of the following.

- (i) State and prove Sandwich theorem of sequence. 5
- (ii) If $(x_n) \rightarrow p$ and $(y_n) \rightarrow q$ then prove that $(x_n - y_n) \rightarrow p - q$. 5
- (iii) Prove that the sequence $(\frac{1}{n^2}) \rightarrow 0$. 5
- (iv) Find limit superior and limit inferior of the sequence $(-1)^n$. 5
- (v) If $\lim_{x \rightarrow a} f(x) = l$ then prove that for any sequence $(x_n) \rightarrow a$, the sequence $f(x_n) \rightarrow l$. 5
- (vi) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is given by
 $f(x) = 1$ if $x \in \mathbb{Q}$
 $f(x) = -1$ if $x \in \mathbb{R} \setminus \mathbb{Q}$
Prove that f is not continuous. 5

P.T.O.

Q.3

Attempt any three of the following.

- (i) Find fourier series of $f(x) = x$ for $x \in [-\pi, \pi]$. 5
- (ii) If $\sum x_n$ converges and $\sum y_n$ converges then prove that $\sum(x_n - y_n)$ converges. 5
- (iii) If $\sum x_n$ converges then prove that $\lim x_n = 0$. Is the converse true? Justify. 5
- (iv) Discuss the convergence of the series $\sum \frac{n^2}{n^5+2}$. 5
- (v) State and prove limit form of ratio test. 5
- (vi) Discuss the convergence of the series $\sum \frac{1}{n^n}$. 5

Q.4

Attempt any three of the following

- (i) Prove that union of two closed sets in \mathbb{R} closed in \mathbb{R} . 5
- (ii) Prove that the finite set do not have any limit point. 5
- (iii) Prove that every convergent sequence is Cauchy sequence. 5
- (iv) Prove that the sequence $\left(a^{\frac{1}{n}}\right) \rightarrow 1$ for $a > 0$. 5
- (v) Prove that the series $\sum \frac{1}{n(n+1)}$ converges. 5
- (vi) State and prove comparison test for convergence of series. 5
