

**B. N. BANDODKAR COLLEGE OF SCIENCE – THANE**  
**F.Y.B.Sc. Theory Examination March 2015**  
**Semester II**  
**USMT 201**

DURATION:  $2\frac{1}{2}$  HOURS

MARKS :75

N.B.: 1 All questions are compulsory.  
 2 figures on right side indicates max. marks.

- Q.1 A) Attempt any One. (8)**
- (i) State & prove Alternating series test. (8)
- (ii) p) Let  $\sum a_n$  &  $\sum b_n$  are convergent series converging to A & B respectively, then prove that  $\sum(a_n + b_n)$  converges to A+B. (4)
- q) Discuss the convergence of the series  $\sum_{n=1}^{\infty} \frac{n}{n^2 - \cos^2 n}$ . (4)
- B) Attempt any Three. (12)**
- i) Prove that Absolutely convergent series are convergent. (4)
- ii) Discuss the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{n 2^n}$ . (4)
- iii) Find the value of 'a' for which a series  $(1-a) + (a-a^2) + (a^2-a^3) + \dots$  Converges. (4)
- iv) Determine whether a series  $\sum \frac{(-1)^{n+1}}{n!}$  is absolutely convergent or conditionally convergent. (4)
- Q.2 A) Attempt any One. (8)**
- (i) a) State & Prove Intermediate value Property. (8)
- (ii) p) Prove that there exist a real number such that it is 1 less than its cube. (4)
- q) Find  $\frac{dy}{dx}$  for  $e^{x^2} + e^{2y} = e^{x+y}$ , using Implicite differentiation. (4)
- B) Attempt any Three. (12)**
- i) Find Derivative of  $f(x) = x^n, n \in \mathbb{N}$  using definition. (4)
- ii) Check differentiability of function  $f(x) = \sin x + 3 \quad x < 0,$   
 $\phantom{f(x) = \sin x + 3} = x + 3 \quad x \geq 0, \text{ at } x = 0.$  (4)
- iii) If  $|x| > 1$ , then find derivative of  $y = \sec^{-1}x$ . (4)
- iv) Find Derivative of  $y = \sqrt{x^2 + 2x + 3}$ , using Chain rule. (4)
- Q.3 A) Attempt any One. (8)**
- (i) State & prove Rolle's theorem. (8)
- (ii) p) State & prove Lagrange's Mean value theorem. (4)

P.T.O.

q) Verify Cauchy's Mean value theorem for  $f(x) = x^3 - 4x$  &  $g(x) = x^2 + 1$  in  $[0,1]$ . (4)

B) **Attempt any Three.** (12)

i) Verify Rolle's theorem for  $f(x) = x(x - 2)^3$  in  $[0, 2]$ . (4)

ii) Find the intervals on which  $f(x) = x^3 - 12x + 1$ , is increasing & decreasing. (4)

iii) Find the local maximum & local minimum value of the function  $f(x) = 10x^3 - 15x^2 + 10$ . (4)

iv) Find  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x}$  using L'Hospital rule. (4)

Q.4

**Attempt any Three.** (15)

i) Show that  $\sum a_n$  &  $\sum b_n$  are two positive term series such that  $\lim_{n \rightarrow \infty} \left(\frac{a_n}{b_n}\right) = l$ , where  $l$  is non-zero finite no. then two series converge or diverge together. (5)

ii) Prove that  $\sum \frac{1}{n^p}$  converges iff  $p > 1$ . (5)

iii) Prove that Every differentiable function is continuous. (5)

iv) Find  $n^{\text{th}}$  derivative of  $x^2 e^{ax}$ , using Liebnitz rule. (5)

v) For what values of  $x$  is the curve  $y = 3x^2 - 2x^3$ , Concave upwards & Concave downward & find its point of inflection. (5)

vi) Sketch the graph of the function  $f(x) = 2x^3 + 6x^2 - 3$ . (5)

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