B. N. Bandodkar College of Science, Thane

S.Y.B.Sc First Term End Examination, October-2011

Subject: Mathematics Paper: II

Duration: 2 hours **Total Marks: 60**

Instructions to the candidates:

1. **All questions are compulsory**
2. **Figures to the right indicate marks.**

Q.1 (a) Prove that $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 / 2x_1 + x_2 - x_3 = 0\}$ is a subspace of $\mathbb{R}^3$. **03**

(b) Attempt any three of the following:

(i) Let V be a vector space over $\mathbb{R}$ and W be a non-empty subset of V. Prove that W is a subspace of V if and only if **04**

$$\alpha, \beta \in \mathbb{R} \; and \; x, y \in W \;\Rightarrow\; \alpha x + \beta y \in W$$

(ii) Let V be an inner product space. For $x, y \in V$, show that **04**

$$\| x + y \| \le \| x \| + \| y \|$$

(iii) Prove that $S = \{1, \sin x, \cos x\}$ is an orthogonal set of $V = C[-\pi, \pi]$ where **04**

$$\langle f, g \rangle = \int_{-\pi}^{\pi} f(x) g(x) dx.$$

(iv) Reduce the following matrix to its row echelon form: **04**

$$A = \begin{pmatrix} 2 & -1 & -1 & 2 \\ 1 & -1 & 1 & 2 \\ 3 & -2 & 1 & 5 \end{pmatrix}$$

(v) Express $A = \begin{pmatrix} 5 & -1 \\ 4 & 6 \end{pmatrix}$ as a product of elementary matrices. **04**

Q.2 (a) Find a matrix E such that $EA = B$ where **03**

$$A = \begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix} \; and \; B = \begin{pmatrix} -4 & 2 \\ 1 & 5 \end{pmatrix}.$$

(b) Attempt any three of the following:

(i) Define Invertible matrix. Further, if A is an $n \times n$ invertible matrix, prove that A^T is invertible and $(A^T)^{-1} = (A^{-1})^T$. **04**

(ii) Let A be an $n \times n$ matrix. Show that A can be expressed as a sum of a symmetric and a skew symmetric matrix. **04**

(iii) Define elementary matrix. Prove that elementary matrices are invertible. **04**

(iv) Solve the following system of linear equations using Gauss Elimination method: **04**

$$x + 2y + 3z = 3$$
$$2x + 3y + 8z = 4$$
$$3x + 2y + 17z = 1$$

P.T.O.

(v) Find inverse of A using elementary row operations: **04**

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix}$$

Q.3 (a) Let V be a vector space over $\mathbb{R}$. Let W_1 and W_2 be subspaces of V. Prove that $W_1 \cap W_2$ is a subspace of V. **03**

(b) Attempt any three of the following:

(i) Show that $B = \{(1,0,0), (1,1,0), (1,1,1)\}$ is a basis of $\mathbb{R}^3$. **04**

(ii) If S is a non-empty subset of a vector space V, then prove that $L(S)$ is the smallest subspace of V containing S. **04**

(iii) Let V be a vector space over $\mathbb{R}$ and $\{x_1, x_2, x_3\}$ be a linearly dependent set of V. Prove that one of the vectors x_1, x_2 or x_3 is a linear combination of the remaining vectors. **04**

(iv) Find a basis of $W = \{(x, y, z) \in \mathbb{R}^3 / 3x + y - z = 0\}$ and extend it to a basis of $\mathbb{R}^3$. **04**

(v) Find rank of $A = \begin{pmatrix} 2 & 1 & -2 \\ 1 & 0 & 3 \\ 4 & 1 & 4 \end{pmatrix}$. **04**

Hence determine whether A is invertible or not.

Q.4 (a) Find the angle between p and q where $p(x) = x$ and $q(x) = x^2$ using inner product **03**

$$\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1).$$

(b) Attempt any three of the following:

(i) State and prove parallelogram law. **04**

(ii) Let S be a subset of an inner product space V containing non-zero elements. If S is an orthogonal set, then prove that it is a linearly independent set. **04**

(iii) Use Gram-Schmidt process to find the orthogonal set corresponding to the set $\{(0,1,1), (1,-1,0), (2,0,1)\}$ using usual dot product in $\mathbb{R}^3$. **04**

(iv) Define orthogonal complement $W^\perp$ of a subspace W. Also prove that $W^\perp$ is a subspace of the vector space V. **04**

(v) Check whether the function **04**

$$\langle x, y \rangle = x_1^2 y_1^2 + x_2^2 y_2^2 \ , \ \ where\ x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$$

is an inner product or not.

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