B. N. Bandodkar College of Science, Thane

S.Y.B.Sc (A.T.K.T/FAILURE) Examination

August-2011

Date: **Subject: Mathematics**

Paper: II

DURATION: 3 hours **TOTAL MARKS: 90**

Instructions to the candidates:

1. **All questions are compulsory**
2. **Figures to the right indicate marks.**

SECTION I

Q.1 (a) If A is an m x n matrix and B is an n x p matrix, then show that **07**

$$(AB)^T = B^T A^T$$

where A^T denotes transpose of matrix A.

Verify $(AB)^T = B^T A^T$ for $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$.

(b) Attempt any two of the following:

(i) Let A be an invertible matrix. Show that A can be expressed as a product of elementary matrices. **04**

(ii) Solve the following system of linear equations using Gauss Elimination method: **04**

$$\begin{aligned} 2x - y - z &= 2 \\ x - y + z &= 2 \\ 3x - 2y + z &= 5 \end{aligned}$$

(iii) Define **04**

(a) Symmetric matrix

(b) Skew Symmetric matrix

Further show that every square matrix can be expressed as a sum of a symmetric and a skew symmetric matrix.

(iv) Find inverse of A using elementary row operations: **04**

$$A = \begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{pmatrix}$$

Q.2 (a) Let V be a vectorspace over $\mathbb{R}$. Let S be a subset of V. Define linear span of S. Prove that L(S) is the smallest subspace of V containing S. **07**

(b) Attempt any two of the following:

		(i)	Let W_1 and W_2 be subspaces of a vectorspace V. Prove that $W_1 \cap W_2$ is also a subspace of V.	**04**
		(ii)	Prove that $W = \{(x, y, z) \in \mathbb{R}^3 / 2x + 3y + z = 0\}$ is a subspace of $\mathbb{R}^3$.	**04**
		(iii)	If $\{x, y, z\}$ is a linearly independent set, then prove that $\{x + y, y + z, z + x\}$ is a linearly independent set.	**04**
		(iv)	Show that the vectors (1, 1) and (-1, 2) form a basis of $\mathbb{R}^2$.	**04**
Q.3	**(a)**		Let V be an inner product space. For $x, y \in V$, show that $\parallel x + y \parallel \leq \parallel x \parallel + \parallel y \parallel$	**07**
	(b)		**Attempt any two of the following:**	
		(i)	If u, w are perpendicular, then show that $\parallel u + w \parallel^2 = \parallel u \parallel^2 + \parallel w \parallel^2$.	**04**
		(ii)	Check whether $S = \{\sin x, \cos x\}$ is an orthogonal set on $C[-\pi,\ \pi]$ where $< f, g > = \int_{-\pi}^{\pi} f(x) g(x) dx.$	**04**
		(iii)	Show that $\parallel x + y \parallel^2 + \parallel x - y \parallel^2 = 2(\parallel x \parallel^2 + \parallel y \parallel^2)$ for any $x, y \in V$, where V is an inner product space.	**04**
		(iv)	Apply Gram-Schmidt process to obtain orthonormal set corresponding to $\{(0,1,1), (1,-1,0), (2,0,1)\}$ in $\mathbb{R}^3$.	**04**

SECTION II

Q.4	**(a)**		Define linear transformation. Further, Let $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be a linear transformation defined by $T(x, y) = (x, x + y, y)$ and $S: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear transformation defined by $S(x, y, z) = (x + y, y + z).$ Let $B_1 = \{(1,0), (0,1)\}$ and $B_2 = \{(1,0,0), (0,1,0), (0,0,1)\}$ be standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. Find $[m(T)]_{B_1}^{B_2}$, $[m(S)]_{B_2}^{B_1}$ and $[m(S\ o\ T)]_{B_1}^{B_1}$. Check that $m(SoT) = m(S)m(T)$.	**07**
	(b)		**Attempt any two of the following:**	
		(i)	Let $T: V \longrightarrow W$ be a linear transformation. Prove that T is injective if and only if $\ker T = \{0\}$.	**04**
		(ii)	Let U, V, W be vectorspaces over $\mathbb{R}$. Let $T: U \longrightarrow V$ and S : $V \longrightarrow W$ be linear transformations. Then prove that $SoT: U \longrightarrow W$ is also a linear transformation.	**04**
		(iii)	Prove that $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ defined by $T(x, y) = (x + 5y, y, -x + 3y)$ is a linear transformation.	**04**
		(iv)	Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find ker T and Im T. Find a basis for each of them and their dimensions.	**04**

Q.5	**(a)**		State and prove Cramer's rule.	**07**
	(b)		**Attempt any two of the following:**	
		(i)	Let A be an invertible n x n matrix. Then show that $Det(A^{-1}) = (Det\ A)^{-1}$.	**04**
		(ii)	Find the area of the parallelogram ABCD where $A = (1,1),\ B = (2,-1)$ and $C = (4,6)$ using determinant.	**04**
		(iii)	For the following matrix, find adjoint of A and A^{-1}. $A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$	**04**
		(iv)	Solve the following system of linear equations using Cramer's rule $2x - y + z = 1$, $x + 3y - 2z = 0$, $4x - 3y + z = 2$	**04**
Q.6	**(a)**		Define Eigen value, Eigen vector and characteristic polynomial of a matrix. Further show that similar matrices have same characteristic polynomial.	**07**
	(b)		**Attempt any two of the following:**	
		(i)	Let A be an $n \times n$ invertible matrix. If λ is an Eigen value of A, then prove that $\lambda \neq 0$ and λ^{-1} is an Eigen value of A^{-1}.	**04**
		(ii)	Find characteristic polynomial, eigenvalues and eigenspaces of the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$. Find basis and dimension of each of the eigenspaces.	**04**
		(iii)	Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{pmatrix}$.	**04**
		(iv)	If $A^2 = I$, where I is an n x n identity matrix and A is any n x n matrix, then show that eigenvalues of A are -1 or 1.	**04**

-----------THE END---------