VPM's B. N. BANDODKAR COLLEGE OF SCIENCE, THANE
S.Y.B.Sc. Semester-IV Examination March-2017
Mathematics Paper III (USMT-403)

Duration: $2\frac{1}{2}$Hr **Marks: 75**

Note 1. All questions are compulsory.
2. Figures on right indicate marks.

Q.1 A) Attempt any One. **(8)**

i) Prove that " The Bernoulli Differential Equation is reduces to Linear Differential Equation by the transformation $z = y^{1-n}$ ". Solve $x\frac{dy}{dx} + y = y^3x^{n+1}$.

ii) Solve $(y - x)dx = (y - x + 2)dy$.

B) Attempt any Two. **(12)**

i) Solve $(x + 2y^3)dy = ydx$.

ii) Solve the following differential equation using integrating factor:
$(x^4y^4 + x^2y^2 + xy)ydx + (x^4y^4 - x^2y^2 + xy)xdy = 0$.

iii) Define Homogenous Differential equation. Solve $x(x - y)dy + y^2dx = 0$.

iv) Define Orthogonal Trajectories. Find the orthogonal trajectories of the curve $xy = k\ ,\ k \neq 0$.

Q.2 A) Attempt any One. **(8)**

i) Consider the differential equation $y'' + Py' + Qy = R$ where P, Q are constants and R is a function of x with $C.F = c_1y_1 + c_2y_2$ where y_1 *and* y_2 are solutions of $y'' + Py' + Qy = 0$. Let $P.I = uy_1 + vy_2$ where u and v are functions of x then $u = -\int\frac{y_2R}{W}dx\ ,\ v = \int\frac{y_1R}{W}dx$ and W is Wronskian.

ii) a) Show that "If y_g is the general solution of the equation $y'' + P(x)y' + Q(x)y = 0$ and y_p is any particular solution of the equation $y'' + P(x)y' + Q(x)y = R(x)$, then $y_g + y_p$ is the general solution of $y'' + P(x)y' + Q(x)y = R(x)$".

b) Define Wronskian. Determine whether the following functions are linearly independent or not where $y_1 = x^4,\ y_2 = x^4 \log x$

B) Attempt any Two. **(12)**

i) Solve the following differential equation by method of Undetermined Coefficients.
$(D^2 - 1)y = e^x \sin x$.

ii) Solve the following differential equation by method of Variation of Parameters.
$y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$

iii) Solve the IVP $4y'' - 4y' + y = 0, y(0) = 1\ , y'(0) = -1.5$.

iv) Let $y_1(x)$ *and* $y_2(x)$ be any two solutions to the differential equation $y'' + Py' + Qy = 0$ on $[a, b]$, then their Wronskian $W(y_1, y_2)$ is either identically zero or never zero.

Q.3 A) Attempt any One. **(8)**

i) Evaluate y(0.4) using Milne-Simpson's method for differential equation $\frac{dy}{dx} = 3 - 5y$ with $y(0) = 1$, $y(0.1) = 1.266$, $y(0.2) = 1.467$, $y(0.3) = 1.613$ correct upto 4-decimal places.

ii) Use Runge-Kutta method of 4^{th} order to find y(1) & y(2) for differential equation $\frac{dy}{dx} = xy$ with y(0) = 1.

B) Attempt any Two. **(12)**

i) Find the value of $y_4(0.5)$ for differential equation $y' = x^2 + y$ with y(0) = 0 by Picard's method, correct upto 4-decimal places.

ii) Evaluate y(2) using Polygon method with h = 0.2 for differential equation $y' = -2x - y$, $y(0) = 1$.

iii) Find the value of y at x = 0.5 for differential equation $y' = x^2 + 3y^2$ with y(0) = 1 by taking the first four terms of Taylor's series expansion, correct upto 4-decimal places.

iv) Evaluate y(2) using Heun's method with h = 0.5 for differential equation $y' = \frac{2y}{x}$, with y(1) = 2.

Q.4 Attempt any Three. **(15)**

i) Solve $\frac{dy}{dx} = sin^2(x - y + 1)$.

ii) Define Exact Differential Equation. Solve $\frac{dy}{dx} = -\frac{4x^3y^2 + y\cos xy}{2x^4y + x\cos xy}$

iii) Find the general solution of $y'' - \frac{x}{x-1}y' + \frac{1}{x-1}y = 0$ given that $y_1 = x$ is a solution.

iv) Define Linearly dependent functions. Solve $y'' + 3y' + y = 0$.

v) Use Runge-Kutta method of 2^{nd} order to find y(0.1) & y(0.2) for differential equation $y' = x - y$ with y(0) = 1.

vi) Evaluate y(0.2), y(0.4), y(0.6), y(0.8) using Eulers method with h = 0.2 for differential equation $y' = 1 - y$, y(0) = 0 correct upto 4-decimal places.

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P.T.O.