V.P.M.'S B. N. Bandodkar College of Science, Thane
S.Y.B.Sc. semester IV Theory Examination March 2017
USMT 401

Duration: 2 Hours 30 min. **Max. Marks :75**

N.B.: 1. **All questions are compulsory.**
2. **Figures to right indicates maximum marks.**

Q.1 A) Attempt any One. **(8)**

i) If $f: S \subseteq \mathbb{R}^n \to \mathbb{R}$ is a scalar field, $a \in S, u \in \mathbb{R}^n$ and $a + tu \in S, \forall\, 0 \le t \le 1$. Suppose $D_u f(a)$ exists for all $a + tu$ where $0 \le t \le 1$. then show that there exist $\theta, 0 < \theta < 1$ and $f(a+u) - f(u) = D_u f(a + \theta u)$. Use it to find the value of θ, for $f(x,y) = x^2 + y$, $a = (1,0)$, $u = (-1,1)$

ii) If $f, g : S \subseteq \mathbb{R}^n \to \mathbb{R}$ are scalar fields, $a \in S$ then show that if $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$ then $\lim_{x\to a}(f+g)(x) = l + m$ and $\lim_{x\to a}(f.g)(x) = l.m$

B) Attempt any Two. **(12)**

i) Show that $S = \{ (x,y) \epsilon\, \mathbb{R}^2 /\ x + y < 2\}$ is an open set.

ii) Show that the limit of function $f(x,y)$ does not exist at $(0,0)$ for

(i) $f(x,y) = \frac{x^2y}{x^4+y^2}$ $if\ (x,y) \neq (0,0)$
$= 0$ otherwise

(ii) $f(x,y) = \frac{xy}{|xy|}$ $if\ xy \neq (0,0)$
$= 0$ otherwise

iii) Show that ,the limit of a scalar function at a point if exist, is unique.

iv) If $f: S \subseteq \mathbb{R}^n \to \mathbb{R}^m$ be vector field, $a \in S$, then show that f is continuous at a if and only if each component function f_i $(1 \le i \le m)$ is continuous at a.

Q.2 A) Attempt any One. **(8)**

i) If $f: S \subseteq \mathbb{R}^n \to \mathbb{R}$ is scalar field, differentiable at $a \in S$ then show that for any $u \epsilon\, \mathbb{R}^n$, the directional derivative of f at a in the direction u, $D_u f(a)$ exist and $D_u f(a) = Df(a)$. Also find directional derivative of $f(x,y) = 3x^2 + 9xy$ at $(1,-1)$ in the direction $(-2,3)$.

ii) If $f: S \subseteq \mathbb{R}^2 \to \mathbb{R}$ is scalar field, $(a,b) \in S$ such that f_x, f_y and f_{xy} exist in some neighbourhood of (a,b) and if f_{xy} is continuous at $(a.b)$ then show that f_{yx} exist at (a,b) and $f_{yx}(a,b) = f_{xy}(a,b)$.

B) Attempt any Two. **(12)**

i) State and prove Chain rule for scalar field.

ii) If $f, g : S \subseteq \mathbb{R}^n \to \mathbb{R}$ are scalar fields, such that ∇f and ∇g exist at $a \in S$ then show that $\nabla(f+g)$ exist at a, $\nabla(f+g)(a) = \nabla f(a) + \nabla g(a)$ and $\nabla(\frac{f}{g})$ exist at a, $\nabla\left(\frac{f}{g}\right)(a) = \frac{g(a)\nabla f(a) - f(a)\nabla g(a)}{(g(a))^2}$ provided $g(a) \neq 0$.

iii) Evaluate f_{xy} *and* f_{yx} if exist at (0,0) and check whether they are equal for

$f(x,y) = \frac{x^2y\,(x-y^3)}{x^2+y^2}$ $if\ (x,y) \neq (0,0)$
$= 0$ otherwise

iv) Find the equation of tangent plane and normal line to the surface
$4xy = 3xy^2 + 6y^3z + 7xz^2$ at $(-1,1,1)$.

(P.T.O.)

Q.3 A) Attempt any One. (8)

i) State and prove Taylor's formula for a function of two variable..

ii) Find local maxima, local minima and saddle point of function

$$f(x,y) = x^2 + xy + y^2 + x + y.$$

Find the values of $f(x,y)$ at these points. Also find Hessian matrix of these point.

B) Attempt any Two. (12)

i) Find maximum and minimum values of function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$ using Lagrange's multiplier.

ii) Find the expansion of the function $f(x,y) = x^4 + x^2y^2 - y^4$ at (1,1) using Taylor's formula of two variables upto second degree.

iii) Find $D\, g(1,1), D\, f(g(1,1))$ and $D\, f_o g(1,1)$ for vector field functions given by $f: \mathbb{R}^2 \to \mathbb{R}^2, g: \mathbb{R}^2 \to \mathbb{R}^2, f(u,v) = (uv, u^2 + v^2)$ and $g(x,y) = (x^2 + y^2, 2xy)$. Also show that $Df_o g(1,1) = D\, f(g(1,1)).D\, g(1,1)$.

iv) Find $\frac{\partial \omega}{\partial s}$ and $\frac{\partial \omega}{\partial t}$ for $\omega = xy + yz + zx$, where $x(s,t) = e^{st}, y(s,t) = t^2$, $z(s,t) = s^2t^2$ at $s = 1, t = -1$ using chain rule.

Q.4 Attempt any Three. (15)

i) Find partial derivatives of the function $f(x,y) = x^2 + xy + y^2$ at (1,2), using definition.

ii) If $f: \mathbb{R}^2 \to \mathbb{R}, f(x,y) = x + 2y^2$ then show that $\lim_{(x,y)\to(1,1)} f(x,y) = 3$ using $\epsilon - \delta$ definition.

iii) Find $\frac{dz}{dt}$, for $z = z(x,y) = 2\sin xy + \cos y$, where $x = t^3, y = t^2$ using chain rule.

iv) Find total derivative of $f: \mathbb{R}^2 \to \mathbb{R}, f(x,y) = x^3 + y$ at (1,2) using definition.

v) Find Jacobian matrix for $f: \mathbb{R}^2 \to \mathbb{R}^2$ and $g: \mathbb{R}^2 \to \mathbb{R}^3$ given by

$f(x,y) = (x^2y + y^2x, e^{-xy})$ at (1,0) and

$g(x,y) = (2x^2 + 3y, 4x - 2y, x^3 + y^3)$ at (1,1).

vi) If $f: S \subseteq \mathbb{R}^n \to \mathbb{R}^m$ be vector field, $f = (f_1, f_2, f_3 \dots\dots f_m)$, $a \in S$, then show that f is differentiable at a if and only if each f_i is differentiable at a and $Df(a)(u) = (Df_1(a)(u), Df_2(a)(u), Df_3(a)(u) \dots\dots, Df_m(a)(u))$

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