B.N.Bandodkar College Of Science, Thane.
Preliminary Examination (Feb.2012)
T.Y.B.Sc. Statistics I.

Duration: 3 hrs. Total Marks:100

N.B.
1. All questions are compulsory.
2. Attempt any **FOUR** sub questions from question **ONE** and any **TWO** sub questions from remaining questions.
3. Figures to right indicate full marks.
4. Use of calculator is allowed.

Q.1

a) Derive the expression for probability that no cell remains empty when 'r' indistinguishable balls are put in 'n' cells. (5)

b) Define i) Generating Function (GF) ii) Probability Generating Function (PGF) iii) Convolution of two sequences. (5)
Obtain PGF of binomial variate.

c) Define order statistics based on a random sample of size n. (5)
Derive distribution of first order statistic.

d) With reference to Queuing theory, explain i) Transient State and Steady State Probabilities.ii) Customer's behavior. (5)

e) If (X,Y) $\rightarrow$ BVN(μ_1,μ_2,1,1, ρ), then, (5)
x) State probability mass function.
y)Derive marginal distribution of Y.

f) Explain Fisher's Z transformation. Hence derive the significance test to test hypotheses $\rho_1 = \rho_2$ in usual notations. (5)

Q.2

a) State theorem on probability of realization of i) at least one event ii) exactly m events out of N events. Hence prove the theorem on probability of realization of at least m events, where $1 \le m \le N$. (10)

b) i) Prove that negative binomial distribution is k fold convolution of geometric distribution. (5)
ii) If $P_X(s)$ is probability generating function (pgf) of random variable X, find pgf of i) Y= $\frac{X-a}{h}$,where a and h are constants. ii) P(X<n) (5)

c) i) In an arrangement of r_1 α's and r_2 β's , what is the probability of having a) 'n_1' α runs and 'n_2' β runs? b) 'k' runs of either kind? (5)
ii) Let X be number of trials required to obtain first success. Find p.g.f. of X. Hence obtain mean of X. (5)

Q.3

a) Derive cumulative density function (c.d.f.) and Hence probability density function (p.d.f.) of r^{th} order statistics. (10)
Stating the postulates for linear growth model, derive the difference (10)

b) differential equations for the same. Obtain the mean number of members in the population at time t, assuming that the initial population size at t=0 is ‘a’

c) A birth process assumes that the system starts with ‘a’ members at time t=0, with no departures allowed. Arrivals occur at the rate of $\lambda_n = n\lambda$ members per unit of time. Obtain the expression for $P_n(t)$, the probability of n members in the system at time t. Hence state average number of members in the system at time t. (10)

Q.4 a) i) For (M/M/1):(FCFS/∞/∞) queuing model obtain distribution of waiting time in the system. (5)

ii) For machine servicing model (M/M/R): (GD/K/K), R<K, obtain an expression for P_n. the probability of n machines being broken down. Also obtain an expression for effective arrival rate. (5)

b) Assuming the expression for P_n, the probability of n customers in the system at time t for general Poisson Queue model, obtain expression for P_n, for (M/M/C): (GD/∞/∞) queuing model. Also obtain expression of i) average number of customers in the system ii) Probability that there are ‘c’ customers in the system. (10)

c) Assuming the expression for steady state probabilities P_n for general Birth and Death process, obtain an expression for P_n, the probability of n members in the system for (M/M/1): (FCFS/N/∞) Queueing model. Hence obtain expression for L_q, average number of members in a queue. (10)

Q.5 a) Derive the distribution of sample correlation coefficient when population correlation coefficient is zero. (10)

b) If (X,Y) follow Trinomial distribution, obtain (10)

i) m.g.f. of (X,Y).

ii) Conditional distribution of X/Y=y.

iii) Distribution of X+Y

c) If (X,Y) $\rightarrow$ BVN($\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$), (5)

i) Obtain marginal distribution of Y using joint distribution of X and Y

ii) State joint m.g.f. Hence obtain joint distribution of $U = \frac{X-\mu_1}{\sigma_1} and v = \frac{y-\mu_2}{\sigma_2}$ (5)

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