B. N. BANDODKAR COLLEGE OF SCIENCE, THANE
S.Y.B.Sc. THEORY EXAMINATION MARCH-2015
USMT 403

DURATION: $2\frac{1}{2}$ HOURS MARKS: 75

N.B. 1 All questions are compulsory.
2 Figures on right indicates marks.

Q.1 A) Attempt any One. (8)

(i) Show that $\int_a^b \frac{1}{(b-x)^p}\,dx$ converges iff $p < 1$. Discuss the convergence of $\int_1^2 \frac{1}{\sqrt{2-x}}\,dx$.

(ii) p) The arc of the parabola $y = x^2$ from $y = 1$ to $y = 4$ is rotated about Y-axis. Find the surface area of resulting solid.

q) Find the length of the parametric curve $x = 3t - t^3, y = 3t^2, 0 \le t \le 2$.

B) Attempt any Three. (12)

i) Compute the following integral by definition $\int_{-2}^{2} \frac{1}{\sqrt{4-x^2}}\,dx$.

ii) Show that the volume of sphere of radius r is $\frac{4}{3}\pi r^3$.

iii) Discuss the convergence of $\int_0^\infty \frac{x}{(1+x)^4}\,dx$.

iv) Find the area of region between the curves $y = \sqrt{x}$ and $y = \frac{x}{2}$.

Q.2 A) Attempt any One. (8)

(i) Derive the formula for finding approximate root of an equation by Bisection method. Perform 4-iterations of Bisection method to find the approximate root of equation $x^2 - 3 = 0$.

(ii) p) Decompose the given matrix into LU-form by using Cholesky method.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

q) State the Descarte's rule of sign.How many atmost positive and negative real roots the polynomial p(x) has where $p(x) = x^4 + 2x^3 - 5x^2 - x + 10 = 0$.

B) Attempt any Three. (12)

i) Find the root of the equation $f(x) = 3x - \log_{10} x - 6 = 0$ correct upto 4-decimal places by Fixed Point iteration method. Take $x_0 = 1$.

(P.T.O.)

ii) Perform 1-iteration of Newton's method to solve the following system of equations $x^2 - 2x - y + 0.5 = 0$, $x^2 + 4y^2 - 4 = 0$. Take $x_0 = 2$, $y_0 = 0.25$

iii) Perform 2- iterations of Secant method to find the root of the equation $f(x) = x^3 - 5x + 1 = 0$, correct upto 3-decimal places.

vi) Perform 2-iterations of Newton-Raphson method to find the approximate root of the equation $3x - \cos x = 1$. Take $x_0 = 1$.

Q.3 A) **Attempt any One.** (8)

(i) Explain Eulers method. Evaluate y(0.3) using Eulers method with h = 0.1 for d.e. $y' = 1 + xy$, $y(0) = 2$.

(ii) p) Use Runge-Kutta method of 2^{nd} order to find y(0.2) for d.e. $\frac{dy}{dx} = x - y$ where $y(0) = 1$ & $h = 0.1$.

q) Find the value of y at x = 0.1 for d.e. $y' = 1 + xy$ with $y(0) = 2$ by Picard's method, correct upto 4-decimal places.

B) **Attempt any Three.** (12)

i) Evaluate y(2) using Heun's method with h = 0.5 for d.e. $y' = 2xy$, $y(1) = 2$.

ii) Find the value of y at x = 0.5 for d.e. $y' = x^2 + y^2$ with $y(0) = 1$ by Taylor's series method, correct upto 4-decimal places.

iii) Evaluate y(0.4) using Milne's-Simpson method for d.e. $\frac{dy}{dx} = x - y^2$ with $y(0) = 0$, $y(0.1) = 0.01$, $y(0.2) = 0.03$, $y(0.3) = 0.0599$, $y(0.4) = 2.1621$, correct upto 4-decimal places.

iv) Evaluate y(2) using Polygon method with h = 1 for d.e. $y' = 2x + y$, $y(0) = 1$.

Q.4 **Attempt any Three.** (15)

i) Find the area under the curve $y = \frac{1}{1+x^2}$ from $x = -\infty$ to $x = \infty$.

ii) Define Gamma function. Show that for $n > 0$,
(a) $\Gamma(n+1) = n\,\Gamma n$. (b) $\Gamma(n+1) = n!$.

iii) Solve the following system of equations by Doolittle's method
$2x + 3y - 4z = 1$, $5x + 9y + 3z = 13$, $-18x - 2y + z = -9$;

iv) Show that the rate of convergence of Newton-Raphson method is quadratic.

v) Use Runge-Kutta method of 4^{th} order to find y(0.2) for d.e. $\frac{dy}{dx} = 1 + y^2$ where $y(0) = 0$ & $h = 0.2$.

vi) Evaluate y(2) using Adam's- Bashforth Moulton method for d.e. $\frac{dy}{dx} = \frac{2y}{x}$ with $y(1) = 2$, $y(1.25) = 3.13$, $y(1.5) = 4.50$, $y(1.75) = 6.13$. Correct upto 4-decimal places.

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