

# B. N. BANDODKAR COLLEGE OF SCIENCE, THANE

F.Y.B.Sc. (INFORMATION TECHNOLOGY) SEMESTER – I EXAMINATION; OCTOBER 2014

COURSE CODE– USIT102

Duration: 2½ Hrs

Total Marks: 75

N.B. 1. All questions are compulsory.

**Q. 1** Answer any two out of following **10**

1. If  $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ , show that  $\text{Adj } A = 3A^T$ .
2. Show that every square matrix can be uniquely expressed as the sum of a Hermitian matrix and a skew-Hermitian matrix.
3. Show that,  $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$  is orthogonal matrix.
4. Discuss the consistency of the system and if consistent solve the equations:  
 $4x - 2y + 6z = 8$      $x + y - 3z = -1$      $15x - 3y + 9z = 21$

**Q. 2** Answer any two out of following **10**

1. Determine Algebraic multiplicity of matrix  $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$
2. Show that given matrix is not diagonalizable,  $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$
3. Apply Cayley-Hamilton theorem to  $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$  and show that  $A^8 = 625 I$ .
4. Examine whether the given vector is linearly independent or dependent  
 $[1, 1, -1]$ ,  $[2, 3, -5]$ ,  $[2, -1, 4]$

**Q. 3** Answer any two out of following **10**

1. If  $\vec{A} = \nabla(xy + yz + zx)$ , find  $\nabla \cdot \vec{A}$ .
2. Find  $\text{curl}(\text{curl } \vec{A})$  where  $\vec{A} = x^2 y \hat{i} - 2xz \hat{j} + 2yz \hat{k}$  at point  $(1, 0, 2)$
3. Show that,  $A = 3y^4 z^2 \hat{i} + 4x^3 z^2 \hat{j} - 3x^2 y^2 \hat{k}$  is solenoidal.
4. If  $\vec{F} = \text{grad}(xy + yz + zx)$ , find  $(\text{curl } \vec{F})$ .

**Q. 4** Answer any two out of following **10**

1. If the slope of curve is  $(x^2 + 2x + 1)$  Find its equation passes through the point  $(1, 1)$ .
2. Solve,  $(4x + y)^2 \frac{dx}{dy} = 1$ .
3. Solve,  $\frac{dy}{dx} = 1 + \tan(y - x)$ .
4. Find particular solution of,  $\frac{dy}{dx} + yx^2 = 0$ , when  $y=1, x=0$ .

**Q. 5** Answer any two out of following **10**

1. Solve the given Linear Differential equation,  $(D^4 - 6D^3 + 13D^2 - 12D + 4)y = 0$
2. Solve,  $(D^2 + 1)^3 (D^2 + D + 1)^2 y = 0$
3. Solve,  $(D^4 + 8D^2 + 16)y = 0$
4. Find root value for  $(D^4 - 6D^3 + 12D^2 - 8D)y = 0$

**Q. 6** Answer any two out of following **10**

1. Find  $n^{\text{th}}$  derivative for  $y = \sin^3 x$ .
2. Verify Lagrange's Mean Value Theorem for  $f(x) = x^{2/3}$  in  $[-8, 8]$ .
3. Find the absolute maximum and minimum for  $f(x) = x^3$  in  $[-2, 2]$
4. Find  $y_n$  if  $y = \frac{x^n - 1}{x - 1}$ .

**Q. 7** Answer any three out of following **15**

1. Find the matrix A if,  $\begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 7 \end{bmatrix}$
2. Verify Cayley-Hamilton theorem and find  $A^{-1}$  where,  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$
3. Show that,  $\nabla(f \pm g) = \nabla f \pm \nabla g$ .
4. Solve,  $\sqrt{1 - y^2} dx - \sqrt{1 - x^2} dy = 0$ .
5. Find solution for  $(D^3 - 2D^2 - 5D + 6)y = 0$  where  $y(0) = 0, y'(0) = 0, y''(0) = 1$
6. Verify Lagrange's Mean Value Theorem for  $f(x) = \log x$  in  $[1, e]$ .