B.N.Bandodkar College of Science,Thane

F.Y.B.Sc First Semester End Examination Oct. 2011.

USMT 102- Additional

Duration:2 hours **Max.Marks:60**

Instructions to the candidates:

1.All questions are compulsory.

2.Figures to the right indicate full marks.

Q.1 (a) If p is a prime number such that $p \mid a$ and $p \mid (a^2 + b^2)$,then prove that $p \mid b$. (3)

(b) Attempt any three of the following questions

(i) State and prove Euclid's lemma. (4)

(ii) Prove that $\sqrt{13}$ is not a rational number. (4)

(iii) Construct Pascal's triangle upto n=6. (4)

(iv) Prove that,for any positive integer n, n(n+1) is divisible by 2. (4)

(v) State and prove the Binomial Theorem for n ∈ □ ,using the principle of induction. (4)

Q.2 (a) Define the Cartesian product of two sets.Find A x B,where A={a,b,c} and B={1,2}. (3)

(b) Attempt any three of the following questions

(i) State addition and multiplication principles of counting. (4)

(ii) On □ ,define the binary operation * as a*b = a + 2b, for all a,b ∈ □
Check whether * is commutative and associative. (4)

(iii) If 5 points are chosen in an equilateral triangle of side 2.Show that at least two points are at a distance of atmost one unit from each other. (4)

(iv) If X is a finite set and $f:X \to X$ is a function such that $f^2(x) = x$, for $\forall$ x ∈ X .Show that f is a bijection. (4)

(v) Show that f:□ → □ given by f(x) = 3x+ 2 is a bijection and hence find its inverse. (4)

Q.3 (a) Define congruence relation modulo n,with an example. (3)

P.T.O.

(b) Attempt any three of the following questions

(i) Define Euler's ϕ function .Find ϕ(20) and ϕ (7). (4)

(ii) Prove that two integers a and b are congruent modulo a positive integer n if and only if they leave the same remainder when divided by n. (4)

(iii) Find the smallest positive integer modulo 8 ,to which $3^2.3^3.3^4.3^{10}$ is congruent. (4)

(iv) Verify Wilson's Theorem for p =7. (4)

(v) Show that every four digit integer of the type abba, is divisible by 11. (4)

Q.4 (a) Prove that $2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n$, where n is non-negative integer. (3)

(b) Attempt any three of the following questions

(i) Prove that the number of primes is infinite. (4)

(ii) If X and Y are finite sets with $|X|$ = n and $|Y|$ = m.Prove that the number of functions from X into Y is m^n. (4)

(iii) If a $\equiv$b(mod n) and c $\equiv$ d(mod n) then prove that ac $\equiv$ bd(mod n). (4)

(iv) If m and n are two integers such that (m,n) = 1,then prove that $\phi(mn) = \phi(m)\phi(n)$. (4)

(v) If f: □ $\rightarrow$ □ , g: □ $\rightarrow$ □ and f(x)=2x+3 ,g(x)=x^2+2.Find gof and fog. (4)

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