B.N.BANDODKAR COLLEGE OF SCIENCE – THANE

T.Y.B.Sc .PRELIMINARY EXAMINATION -2011

MATHEMATICS PAPER – I

Duration: 3 Hrs Max.Marks 100

N.B. 1. All Questions are compulsory.

Q.1 Attempt any four of the following:

(i) If f: [a,b]→IR is IR-Integrable on [a,b] and a<c<b,then prove that f is IR-Integrable on [a,c] and [c,b]. [5]

(ii) Find Directional derivative of $f(x,y,z)=z^2-x^2-y^2$ at (1,0,1) in the direction of (4,3,0). [5]

(iii) Use Divergence Theorem to evaluate $\iint_s (2x+2y+z^2)ds$ where S is the sphere $x^2+y^2+z^2=1$. [5]

(iv) Determine the Second Order Taylor's Formula for $f(x,y)=(x+y)^2$ at (0,0).

[5]

(v) Let w=xy+yz+zx, x(s,t)=st, $y(s,t)=e^{st}$,$z(s,t)=t^2$.Find dw/dt,dw/ds.

[5]

(vi)Compute $\iint_s xydS$, where S is the surface of the tetrahedron with sides z=0, y=0, x=0, and z=1-x-y. [5]

Q.2 Attempt any two of the following.

(a) State and prove First Fundamental Theorem of Integral Calculus.

[10]

(b) Prove that a continuous function on[a,b] is IR-Integrable. Give an example of an IR-Integrable function which is discontinuous at all rational numbers. [10]

(c) Let f: [a,b]->IR be bounded. Let P_1 and P_2 be the partition of [a,b]such that P_1 is contained in P_2.Prove that

i) $U(f,p_1)>U(f,p_2)$

ii) $L(f,p_1)<U(f,p_2)$ [10]

Q.3 Attempt any two of the following.

(a) Let (f_n) be a sequence of real valued function defined on D IR. Prove that $(f_n)\rightarrow f$ uniformly if and only if $(M_n) \rightarrow 0$ where $M_n=\sup\{f_n(x)-f(x)/x\epsilon D\}$. [10]

(b)Let $f_n(x)=n^2x/ (1-x)^n$, $x\epsilon(0,1)$. Prove that (f_n) does not converge uniformly. [10]

(c)Find the radius of convergence of power series $\sum f_n(x)$, where $f_n(x) =x^n/n^2$. Determine the interval of convergence. [10]

Q.4 Attempt any two of the following.

(a) (i)Prove that If a Scalar field function has a total derivative then it has a Directional derivative in every direction of unit vector.

(ii)Prove that every differentiable scalar field is continuous. [10]

(b)State and prove chain rule for scalar field function. [10]

(c)State and prove Lagrange’s Mean Value Theorem for Scalar Field.Verify for the function $f(x,y)=x^2-y^2$ at the points (0,0) and (1,1). [10]

Q.5 Attempt any two of the following.

(a) (i)State and Prove Stoke’s Theorem.

(ii)Evaluate $\iint_s F. ndS$, where $F(x, y, z) = (1,1,z(x^2+y^2)^2)$ and S is the surface of the cylinder $x^2+y^2 \leq 1$, $0<z<1$. [10]

(b) (i)Compute $\iint_s xydS$, where S is the surface of the tetrahedron with sides z=0, y=0, x=0, and z=1-x-y. [5]

(ii)Evaluate $\iint_s z\, dS$, where S is the surface $z=x^2+y^2$, $x^2+y^2 \leq 1$. [5]

(c) Verify Divergence Theorem for vector field F(x,y,z)=(3x,xy,2xz)over the region V bounded by the planes x=0,x=1,y=0,y=1,z=0,z=1. [10]