

B.N.BANDODKAR COLLEGE OF SCIENCE – THANE

F.Y.B.Sc First Semester End Examination October – 2011

Code – USMT101

Duration – 2 Hrs

Max. Marks 60

N.B. All questions are compulsory.

- Q.1** (a) Let $S = \{ x \in \mathbb{R} / |5x - 2| < 3 \}$. Prove that S is bounded. (3)
(b) **Attempt any three of the following.**
(i) Solve : $|2 - 3x| > 5$. (4)
(ii) Find $\lim_{x \rightarrow 0} x \sin \frac{1}{x}$. (4)
(iii) Prove that the equation $x^3 - 3x - 1 = 0$ has a root. (4)
(iv) Prove that $\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$. (4)
(v) Sketch the graph of $f(x) = x^2 + 4x + 3$. (4)
- Q.2** (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 2x$. Find the derivative of $f(x)$. (3)
(b) **Attempt any three of the following.**
(i) State and prove Leibnitz theorem. (4)
(ii) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Find $\frac{d}{dx} f^{-1}(x)$. (4)
(iii) Find $\frac{dy}{dx}$ if $x^2 + y^2 = x^2(x - y)^2$. (4)
(iv) Give an example of a function which is differentiable once but not twice, justify. (4)
(v) Find n^{th} derivative of $f(x) = \frac{1}{(x-1)(x-3)}$. (4)
- Q.3** (a) Find $\lim_{x \rightarrow 0} \frac{\log(1 + x \sin x)}{\cos x - 1}$. (3)
(b) **Attempt any three of the following.**
(i) Verify Lagrange's Mean Value Theorem for $f(x) = x^2 + 2x - 1$, $x \in [0, 1]$. (4)
(ii) State and prove Cauchy's Mean Value Theorem. (4)
(iii) Find the interval in which $f(x) = 2x^3 + 8x + 10$ is increasing. (4)
(iv) Find the Taylor's expansion of $f(x) = \log(1 + x)$. (4)
(v) Sketch the graph of $f(x) = x^3 - 3x + 3$. (4)
- Q.4** (a) Prove that $x \leq |x|$. (3)
(b) **Attempt any three of the following.**
(i) Find local maxima / minima of $f(x) = x^4 + 2x^3$. (4)
(ii) Prove that $f(x) = 2x + 1$ is continuous on \mathbb{R} . (4)
(iii) Sketch the graph of $f(x) = \tan x$. (4)
(iv) Prove that the function $f(x) = 3x + 4$ is bounded on $[1, 2]$. (4)
(v) State and prove Sandwich theorem. (4)

