B.N. Bandodkar College of Science, Thane

T.Y. B.Sc Preliminary Examination Feb 2012 ,

Statistics Paper II

Max marks: 100 Duration : 3 hrs

N.B. 1) All Questions are compulsory.

2) Attempt any FOUR sub-questions from question 1 and any TWO sub –questions from the remaining questions.

3) Figures to the right indicate full marks.

Q.1) Attempt any four

1. Explain the following terms with suitable example. i)Unbiasedness ii)Consistency iii) Efficiency. 5
2. With reference to point estimation explain the method of modified minimum chi square.. 5
3. Define i) Loss function i) Risk function iii) Prior distribution iv) Posterior distribution. 5
4. Explain the terms with examples:-
   (i) Crtitical region and its size (ii) Errors in testing of hypothesis 5
5. State the advantages and disadvantages of non-parametric methods. 5
6. Explain sequential probability ratio test. 5

Q.2) a) i) If T is an unbiased estimator of θ then show that $\sqrt{T}$ and T^2 are biased estimation of $\sqrt{\theta}$ and θ^2 respectively 5

ii) If X1 ------ Xn is random sample from Poisson distribution with parameter θ Use Neyman's factorization theorem to obtain sufficient estimator of θ. 5

b) Let X_1 --- Xn be a random sample from a population with p.d.f. 10

$$f(x,\theta) = \frac{2xe^{-x^2/\theta}}{\theta} \qquad x > 0 \quad \Theta > 0$$

$= \quad 0 \qquad\qquad$ o. w.

Obtain CRLB for variance of an unbiased estimator of Θ. Examine whether it is attained .

c) i) If X_1 -----Xn be a random sample from Poisson distribution with parameter m. Obtain an unbiased estimator for i) m ii) m². 5

ii) Test whether minimum variance unbiased estimator for Θ exists 5

$$f(x, \theta) = \frac{1}{\pi(1+(x-\theta)^2)} \qquad -\infty < x < \infty, \qquad -\infty < \theta < \infty$$

Q.3) a) i). State any three important properties of m.l.e 3

ii). Let sample of n observations be taken from a population with p.d.f 7

$$f(x, a, b) = \frac{1}{b} e^{-(x-a)/b} \;;\; a < x < \infty \;,\; -\infty < a < \infty \;,\; b > 0$$

Estimate the parameters a and b by (1) method of moments and (2) method of maximum likelihood

b) i) Define (1) Loss function (2) Risk function 3

ii) Let $X_1, X_2 \ldots\ldots\ldots X_n$ be a random sample from Poisson distribution with parameter θ. The prior distribution of θ is an exponential distribution with mean one. Using squared error loss function, find Bayes' estimate of θ. 7

c) i)Explain the term interval estimation 2

ii) Obtain 100(1-α) % confidence interval for θ , the parameter of a exponential distribution using asymptotic property of maximum likelihood estimators 8

Q4 a)State and prove Neyman-Pearson's lemma for testing simple null against simple alternative hypothesis. 10

b)Obain an uniformly most powerful test for testing Ho: $\lambda = \lambda_0$ against H1: $\lambda > \lambda_0$ where λ is parameter of poisson distribution. 10

c) Obtain likelihood ratio test for testing Ho: $\mu = \mu_o$ against H1: $\mu \neq \mu_o$ where μ is parameter of N(μ , σ^2) distribution, σ^2 is unknown. 10

Q5 a)Explain following non-parametric procedures:- 10

(i) Sign test for two samples

(ii) Run test randomness

b) Obtain sequential probability ratio test of strength (α, β) for testing 10

Ho: $\mu = \mu_o$ against H1: $\mu > \mu_o$ where μ is parameter of N(μ , σ^2) distribution, σ^2 is known.

c) Explain 10

(i) Likelihood ratio test

(ii) Mann-Whitney-Wilcoxon Test

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