B.N.Bandodkar College Of Sc.

Class: T.Y.B.Sc. Subject : Elements of Operations Research

Topic : Dual Simplex Method

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In Simplex method we have seen that if constraints are of $>$ type, we need surplus as well as artificial variables which make calculations tedious.DualSimplexMethod is the technique which deals with only slack variables.

This procedure is similar & opposite to usual simplex method.

	Simplex	Dual Simplex
1	Objective fn is.of minimization (for our convenience)	Objective fn is.of minimization (for our convenience)
2	Inequality & equality constraints	All $\leq$ type inequalities.
3	Starts with feasible & non optimum sol.	Starts with infeasible but(better than) optimum sol.
4	We first decide incoming variable & then using ratio, outgoing variable is decided.	Here outgoing variable is obtained first & then using replacement ratio incoming variable is obtained.
5	Procedure is continued till we get all Z_j-$C_j \leq 0$	We start with all Z_j-$C_j \leq 0$& stop when all entries in column B become non negative.
6	Every problem can be solved using this method.	Some particular problems can be solved using this method.

<u>Prerequisites for Dual simplex Method.</u>

1) Objective fn. Can be minimization or maximization. For our convenience we take it as of minimization.

2) R.H.S.constants are unrestricted in sign.

3) All constraints should be of $\leq$type.

a) If constraint is of $\geq$ type,then multiply throught by -1 & change the inequality to $\leq$type.

b) If the constraint is equality, convert it into two $\leq$type inequalities.As we know that if X=C $\Rightarrow$ X$\leq$C & X$\geq$C$\Rightarrow$ X$\leq$C & -X$\leq$-C

e.g.2x+3y=4 will be converted as 2x+3y$\leq$4 & -2x-3y$\leq$-4

4) Variables unrestricted or restricted to non negativity.

The iterative procedure of Dual Simplex Method.

Steps:

1) Write ob. fn. as of minimization.Write all constraints as of $\leq$ type using third point of prerequisites.[R.H.S. constants can be –ve since the method is applicable to infeasible initial basic sol.]

2) Introduce slack variables & convert $\leq$ inequalities into equalities constraints. Attach cost 0 to all slack variables.Obtain an initial basic sol. &put this sol. in starting dual simplex table.

3) Calculate Z_j-C_j& test the nature of Z_j-$C_j$in starting table.

   a) If all Z_j-C_j $\leq$o &all x_{Bi} (basic variables) are at +ve level then optimum basic feasible sol. has been obtained. Procedure ends.

   b) If all Z_j-C_j $\leq$o & at least one x_{Bi} (basic variable) is –ve go to step 4

   c) If at least one Z_j-C_j is +ve the method is not applicable.(method is applicable to starting infeasible but optimum sol.)

4) Select the most –ve x_{Bi} corresponding basic variable then leaves the basis.Let x_{Bk} =min. x_{Bi} then k^{th} basic variable leaves the basis. Tie for outgoing vector is broken arbitrarily.

5) Test the nature of y_{kj} (k^{th} row elements) j=1,2……..n

   a) If all y_{kj} are non –ve then there does not exist any feasible sol. i.e. problem has no sol.

   b) If at least one y_{kj} is -ve then compute the replacement ratio for non basic variables as

$\{ \frac{Zj - Cj}{y_{kj}}, \ \forall \ y_{kj} < 0\}$ j=1,2…….n & choose the minimum of these ratios. Corresponding column vector say y_r then enters the basis. Tie for incoming vector is broken arbitrarily.

6) Prepare new table as in usual Simplex procedure.

7) Repeat the procedure until either an optimum feasible sol.(all Z_j-C_j $\leq$o &all basic variables are at +ve level) has been obtained or there is an indication of non existence of a feasible sol.

Solution Of Dual from Dual Simplex Method.

1) To read the finite optimum sol. of dual. the technique is same as that of Simplex method. In final table read Z_j entries corresponding to slack variables. If corresponding dual variable is restricted then sol. is $|Z_j|$ & if corresponding dual variable is unrestricted then sol. is $\pm Z_j$ where sign of Z_j is determined using the fact that optimum of primal ob.fn.= optimum of dual ob.fn.

2) If there is no sol. to primal then dual has (a) no sol. with primal objective is to be maximized. (b) Unbounded sol. with primal objective is to be minimized.