B.N.BANDODKAR COLLEGE OF SCIENCE – THANE

S.Y.B.Sc. (A.T.K.T/FAILURE) EXAMINATION – Feb.2011

MATHEMATICS PAPER – I

Duration: 3 Hrs Max.Marks:90

N.B. 1. All Questions are compulsory.

SECTION I

Q.1 (a) Prove that intersection of open sets in IR is open in IR. 3

(b) Attempt any three of the following.

(i) State and prove Haudorff's property. 4

(ii) Prove that [a, b] is closed set. 4

(iii) Let x,y ϵ IR such that x < y. prove that for any z >0 , xz < yz. 4

(iv) State and prove Archimedian Property in IR. 4

(v) Prove that if a supremum exist then it is unique. 4

Q.2 (a) Using ϵ-n_o definition to prove that the sequence $(\frac{1}{n}) \to 0$. 3

(b) Attempt any three of the following.

(i) If (x_n) --> p and (y_n) --> p then prove that $(x_n + y_n)$ --> p + q. 4

(ii) Prove that every convergent sequence is Cauchy sequence. 4

(iii) Find limit superior of sequence (1/n). 4

(iv) Define Limit Superior and Limit Inferior of a sequence. 4

(v) If f and g are continuous at a then prove that f g is continuous at a. 4

Q.3 (a) Find the Radius of Convergence of the series $\sum \frac{x^n}{3n}$. 3

(b) Attempt any three of the following.

(i) Prove that if a series $\sum x_n$ converges then (x_n) → 0. 4

(ii) Find the fourier series of f(x) = x , x ϵ [-π, π]. 4

(iii) Discuss the convergence of the series $\sum \frac{2n+4}{n^3}$. 4

(iv) State and prove the Cauchy Criterion of the Convergence of the series. 4

(v) Prove that the series $\sum \frac{1}{n^2}$ converges . 4

Section II

Q.4 (a) Solve the following differential equation 3

$$\frac{dy}{dx} = \frac{x+2y}{x-2y}$$

(b) Attempt any three of the following.

(i) Find the general solution of y'' - 5y' + 6y=0. 4

(ii) If $y_1 = x$ is a solution of $x^2y'' + xy' - y = 0$.Find the general solution of the differential equation. 4

(iii) Solve : $2y'' - 4y' + 8y = 0$. 4

(iv) Find the Particular solution of $y'' + 10y' + 25y = 14e^{-5x}$. 4

(v) Solve : $y'' + y = 2\cos x$ 4

Q.5 (a) Write triple integral bounded by the xy – plane and $z = 4 - x^2 - y^2$. 3

(b) Attempt any three of the following.

(i) Evaluate the double integral of f(x ,y) = x + y over the rectangle [0,2]×[-1,1]. 4

(ii) Find the volume of the tetrahedron bounded by the planes Y = 0, z = 0, x = 0 & x + y + z = 1. 4

(iii) Evaluate $\iint (x+2y)dxdy$ where D is the region bounded by $y = x^2$ and y = 1 . 4

(iv) Find the Centre of Mass of a thin plate bounded by $x = y^2$ and x = 2y . 4

(v) Find the average value of f(x, y) = x(x + y) on D = [0,1]×[0,1]. 4

Q.6 (a) Evaluate the line integral of f(x, y) = (x, y) over the curve C : $y = x^2$ from (-1, 1) to (1, 1). 3

(b) Attempt any three of the following.

(i) Use line integral to find area of the Ellipse $x^2 / a^2 + y^2 / b^2 = 1$. 4

(ii) Find the potential function of $F(x, y) = (3 + 2xy, x^2 – 3y^2)$. 4

(iii) State Green's Theorem & use it to evaluate the line integral of $f(x, y) = (2x – y + 4, 5y + 3x – 6)$ along the triangle bounded by (0,0), (3, 0), (3, 2). 4

(iv) Find the work done of $f(x, y) = (x^2, y^2)$ over the curve $y=x^2$ from (-1,1) to (1,1). 4

(v) Find the Flux density of $f(x, y) = (x^2-y, xy - y^2)$. 4