B.N.BANDODKAR COLLEGE OF SCIENCE – THANE

F.Y.B.Sc. (A.T.K.T/FAILURE) EXAMINATION – Aug.2011

MATHEMATICS PAPER – I

Duration : 3 Hrs                                                                 Max. Marks : 90

N.B. 1. All Questions are compulsory.

Section – I

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|---|---|---|---|---|
| Q.1 | (a) | | Prove that $\lvert x + y \rvert \le \lvert x \rvert + \lvert y \rvert$, where $x, y \in$ IR. | 3 |
| | (b) | | Attempt any three of the following. | |
| | | (i) | Sketch the graph of $f(x) = x^2 + 5x + 6$ , $x \in$ IR. | 4 |
| | | (ii) | Define Left Hand Limit and Right Hand Limit. | 4 |
| | | (iii) | Prove that the function $f(x) = 2x + 1$ is continuous in IR. | 4 |
| | | (iv) | Use ϵ-δ definition to prove that , $\lim_{x \to 2} 3x + 2 = 8$ | 4 |
| | | (v) | Find $x \in$ IR such that $\lvert 2x - 1 \rvert < 7$. | 4 |
| Q.2 | (a) | | Find n th derivative of $f(x) = (2x + 3)^n$. | 3 |
| | (b) | | Attempt any three of the following. | |
| | | (i) | Let f : IR -> IR be given by $f(x) = 2x^2$. Find the derivative of f at a using definition. | 4 |
| | | (ii) | State and prove Leibnitz rule. | 4 |
| | | (iii) | Prove that every differential function is continuous. | 4 |
| | | (iv) | If f and g are differentiable then prove that f + g is differentiable . | 4 |
| | | (v) | Find $n^{th}$ derivative of $f(x) = x^4e^{3x}$. | 4 |
| Q.3 | (a) | | Find local maxima/minima of $f(x) = 3x^3 + 9x^2$. | 3 |
| | (b) | | Attempt any three of the following. | |
| | | (i) | State and prove Cauchy's Mean Value Theorem. | 4 |
| | | (ii) | Verify Lagrange's Mean Value Theorem for the function f : [1,2] -> IR such that $f(x) = x^2 + 1$. | 4 |
| | | (iii) | Find Taylor's expansion of $f(x) = e^x$ . | 4 |
| | | (iv) | Find the interval in which $f(x) = x^2 + 2x + 1$ is increasing. | 4 |
| | | (v) | Define vertical and horizontal asymptote to the curve. | |

## Section II

Q.4 (a) Find equation of line passing through the point (1 , 2 , -1) and (2, 4, 3). 3

(b) Attempt any three of the following.

(i) Convert the rectangular coordinate (3, 4) into polar coordinate. 4

(ii) Find equation of a plane passing through the point (3, 2, 1) having normal vector (5, 4, 2). 4

(iii) Write spherical equation of $f(x) = x^2 + y^2 + z^2 + 2x - 4$. 4

(iv) Find the angle between the vectors (2, 3, -1) and (1, -2, -3). 4

(v) Find the projection of a vector (5, 3, 12) on (2, 4, 7).

Q.5 (a) Evaluate $\lim_{(x,y)\to(1,1)} \frac{2x+3y}{3x-2y}$ 3

(b) Attempt any three of the following.

(i) 4

$$F(x, y) = \frac{xy}{x^2+y^2} \quad \text{if } (x, y) \neq (0, 0)$$

$$= 0 \quad \text{if } (x, y) = (0, 0)$$

Find limit of f(x, y) along the path y = x as (x, y) →(0, 0) .

(ii) Prove that the following limit doesn't exist. 4

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$$

(iii) Give ϵ - δ definition of limit of function of two variables f(x, y) as (x, y) → (0, 0). 4

(iv) Find $\lim_{(x,y)\to(0,0)} \frac{2x^3}{x^2+y^2}$ using Polar Coordinates. 4

(v) Prove that the function f(x, y) = (2x, 3y) is continuous at (1, 2). 4

Q.6 (a) Find partial derivative of $f(x) = x^3y^2 + y^3x^2$ with respect to x and y. 3

(b) Attempt any three of the following.

(i) Find directional derivative of f(x, y) = 2x + 3y at (1, 2) in the direction of a unit vector (1, 0). 4

(ii) Find all second order partial derivative of $f(x, y) = y^4 - x^2y^2 + 3x^3y$. 4

(iii) Find the gradient of $f(x, y) = x^3 + y^3 + 2xy$. 4

(iv) Find all local maxima and minima of $f(x, y) = x^3 - y^3 - 3xy + 4$. 4

(v) Find minimum value of f(x, y) = x + y subject to xy = 16. 4