

# Memristor: The Advancement in Fundamental Circuit Elements

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## Abstract

For nearly 180 years, circuit theory is studied with three fundamental circuit elements, the resistor, the capacitor and the inductor. These passive two terminal elements describe the relations between voltage, current, charge and flux but do not store information. On this back ground, in 1971 Prof. Leon .O. Chua presented a revolutionary concept of the existence of the fourth missing element called “Memristor” in the list of three basic electrical circuit elements. Introduction of the nano scale Memristor (Resistor with Memory) that can store information without the need of a power source is a paradigm change in fundamental circuit elements [1, 2, 3]. Memristor has tremendous potential to replace conventional flash memories in near future. Its emerging applications include a wide range from analog /digital computation circuits to neuromorphic circuits. Introduction of non linear Memristor is enhancing the quantum computing processing and understanding of biological processes [4]. Due to its novel characteristics and diversifying application potentials, nano scale Memristor is getting a lime light focus from researchers as well as from industry. This paper will discuss the fundamental properties, basic device model and prominent application areas of Memristor as an advancement in fundamental circuit element.

**Keywords:** Memristor, non volatile memory, Hysteresis, Resistance switching, V-I characteristics

## 1. Introduction

In circuit theory, for nearly 180 years, it has been accepted that there are three fundamental passive circuit elements, the resistor, the capacitor and the inductor discovered in 1827, 1745 and 1831 respectively. These passive two-terminal devices do not store information. They describe the relations between voltage, current, charge and flux [1-2]. On this back ground, the paper of Prof. Leon. O. Chua, in 1971 titled “Memristor - The Missing Circuit Element” was revolutionary in the electric circuit theory. His paper presented a logical symmetry reasons for the existence of the fourth missing element which exhibited a relationship between flux ( $\phi$ ) and electric charge ( $q$ ). This element was named as **Memristor**, that is, resistor with memory (concatenation of Memory Resistor). It is essentially a charge dependent resistor which remembers the charge that has flowed through it or the voltage that was last applied across it.

### 1.1 Basic Circuit Laws

Resistance relates voltage and current ( $dv=R.di$ ), capacitance relates charge and voltage ( $dq=C.dv$ ) and inductance relates flux and current ( $d\phi =L.di$ ), respectively as shown in flow circuit (Figure 1).

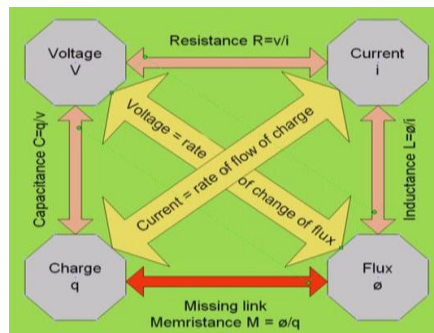


Figure 1: Fundamental circuit element relations

On the application of the Kirchoff's circuit laws, the circuit dynamics is as given in equations below (1 to 5)

Definition of current:  $dq = i dt$  ..... (1)

Faraday's law:  $d\phi = v dt$ .....(2)

Resistor:  $dv = R di$  .....(3)

Capacitor:  $dq = Cdv$  .....(4)

Inductor:  $d\phi = L di$  .....(5)

On the basis of symmetry and through an 'axiomatic approach' Leon Chua established a non linear memristor whose magnetic flux  $\phi$ , is a function of charge that has flown through it and it is defined by

$$d\phi = M dq \text{ .....(6)}$$

Memristance  $M(q)$  is the rate of change of magnetic flux with respect to charge ,

$$M(q(t)) = \frac{d\phi/dt}{dq/dt} = \frac{v(t)}{i(t)} \quad (7)$$

Memresistor characterizing equations (1), (2), (6) & (7) can be rearranged as equation (10) for representing a charge controlled memristor as

$$V = M(q) \cdot i \text{ .....(7)}$$

and a flux controlled memristor as

$$i = W(\phi) \cdot v \text{ ..... (8)}$$

Where  $q = \int_{-\infty}^t i dt$  and  $\phi = \int_{-\infty}^t v dt$  and two nonlinear functions  $M(q)$  and  $W(\phi)$  are termed as memristance and memductance, respectively .

## 2. Memristor Analogy

The simple analog of Memristor in day to day life is explained with an electrolytic cell[4]. Such cell has an internal resistance but when a current passes through it, the chemical reactions inside the cell affect the electrolyte concentration. This makes the resistance of the cell as the function of the current passing through the cell electrodes. When the current is removed, the resistance of the cell is 'Frozen'. Similar behavior of a charge controlled Memristor can be discussed with pipe analogy.

### 2.1 Memristor Pipe Analogy

For an electrical resistor a common analogy is a pipe that carries water. The water is considered analogous to electrical charge; the pressure at the input of the pipe analogous to voltage and the rate of flow of the water through the pipe is considered analogous to electrical current[3,5]. The flow of water through this pipe is faster if the pipe has a larger diameter. An analogy for a memristor is interesting because it is a kind of pipe that expands or shrinks with water flowing through it as shown in Fig.3. If water flows through the pipe in one direction, the diameter of the pipe increases, thus enabling the water to flow faster. If water flows through the pipe in the opposite direction, the diameter of the pipe decreases, thus slowing down the flow of water. If the water pressure is turned off (voltage switched off), the pipe will remember and retain its most recent diameter until the water is turned back on (voltage reapplied). Thus, the pipe does not store water like a bucket (or a capacitor) but it remembers how much water had flown through it.

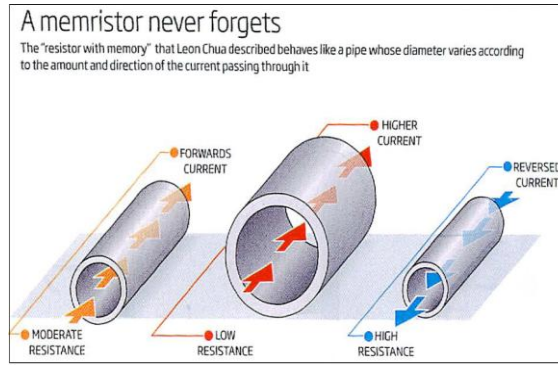


Figure 3 : Memristor Pipe Analogy

### 3. Memristor Properties

As a fundamental circuit element, memristor shows properties like memory effect (non volatile nature), non linearity and hysteresis property. These properties are discussed here in detail.

For a charge dependent memristor equation as expressed in (7) is

$$M(q(t)) = \frac{d\phi/dt}{dq/dt} = \frac{v(t)}{i(t)}$$

If the relationship between  $\phi$  and  $q$  is linear , that is ,  $M(q(t))$  is a constant then  $d\phi/dq = R$  , where  $R$  is constant and thus we obtain a (linear) resistor obeying Ohm's Law i.e.

$$R(t) = v(t) / i(t) \dots \dots \dots (9)$$

If the relationship between  $\phi$  and  $q$  is nonlinear, that is ,  $M(q(t))$  is nontrivial then  $d\phi/dq = M(q)$ , and then we obtain a (non linear) memristor expressed by relation

$$v(t) = i(t)M (q(t)) \dots \dots \dots (10)$$

This equation implies that if  $i(t) = 0$ , we find  $v(t) = 0$  and  $M(t)$  is constant . This is the essence of the **memory effect** (non volatile nature).

As Memristor is not an energy storage device like the capacitor and inductor, the voltage must equal to zero whenever the current goes to zero. This causes the I-V curve of the device to produce a **pinched hysteresis loop** revealing the memory effect associated with it [2] as shown in Figure 4. The reason to call Memristor as a fundamental circuit element is that no combination of non linear resistor, capacitor or inductor can reproduce the hysteresis loop or lissajous behavior of Memristor.

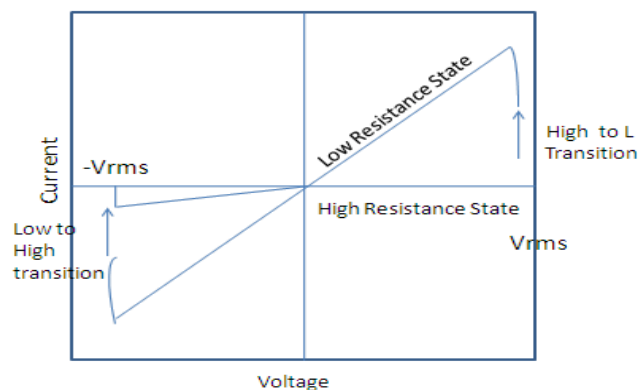


Figure 4 : I-V characteristic (Hysteresis Loop)

It is interesting to observe that a Memristor under AC conditions can switch reversibly between a less conductive OFF state and a more conductive ON state as the voltage polarity of the device changes. For this resistive reversible switching, Memristor is considered as a possible breakthrough in non-volatile memory technology with its resistive switching characteristic.

#### 4. Hp Lab Memristor Model

The memristor was considered nothing more than a mathematical curiosity until the first device and model of a memristor was published in 2008 [3]. Memristor developed by Hp Lab device is comprising of thin conducting  $TiO_2$  layer and insulating  $TiO_{2-x}$  layer sandwiched between two Platinum (Pt) layers. HP Lab research team observed a resemblance in the graph results of Memristor produced [3] and the original predicted model of Leon Chua as shown in Figure 5. It is observed that the pinched hysteresis loop is a function of excitation frequency and is shrunk by increasing the excitation frequency [2,6]. In fact, as the frequency increases towards infinity, the Memristor acts as a linear resistor. After this discovery, there was an extensive research based on the fabrication, modeling and dynamics of titanium dioxide Memristor devices [6,7,8] which is out of scope of this paper discussion.

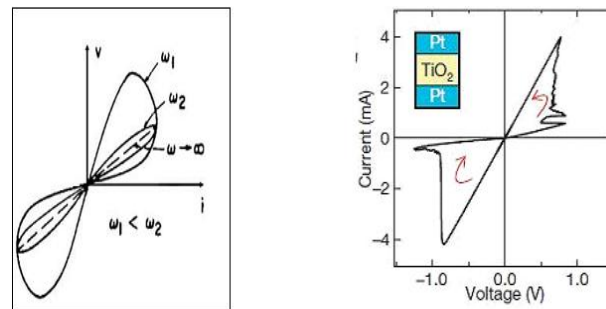


Figure 5: (a) Chua's Predicted Model results and (b) HP Lab Model results

$TiO_2/TiO_{2-x}$  device model of HP Lab comprises of two variable resistors connected in series as shown in Figure 6. Each resistor in the model represents two distinct regions of the metal-oxide [3]. The region with a high ionic dopant concentration is termed  $R_{ON}$  and the other with a low concentration is termed  $R_{OFF}$ . At any instance of time, the static resistance of the Memristor is the sum of the resistances across the doped and undoped regions. Assuming  $w$ , as the length of the high concentration or doped region and that  $w$  is allowed to vary from 0 to  $D$ , the memristance can be modeled as in Figure 6

$$M(w) = R_{ON} * w/D + R_{OFF} * (1 - (w/D)) \dots \dots \dots (11)$$

There can be different values for the memristance  $M$ , as variation in the electric current causes the movement of  $w$  between 0 and  $D$ .

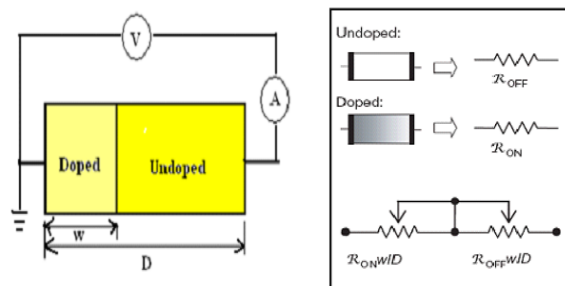


Figure 6 : Memristor Device Structure and Model

The equation for memristance derived by HP Lab researchers [3] is

$$M(q) = R_{OFF} \left( 1 - \frac{\mu_V R_{ON}}{D^2} q(t) \right) \dots\dots\dots (12)$$

where D is the overall thickness of the TiO<sub>2</sub> bi-layer, R<sub>ON</sub> is the resistance when the active region is completely doped (w = D) and R<sub>off</sub> is the resistance when the TiO<sub>2</sub> bi-layer is mostly un doped i.e (w → 0) and μ<sub>V</sub> is the average carriage or doping mobility (m<sup>2</sup>/s/V)

From this equation it is observed that the memristance will be significant for higher doping motilities μ<sub>V</sub> and smaller semiconductor film thicknesses D. Because of the factor 1/D<sup>2</sup> the memristance is more significant at nano scale.

## 5. Applications

The discovery of Memristor as a nanoscale resistance switching device is an extraordinary contribution to fundamental electrical circuits. Now technology has the choice to add Memistor in the old circuit configurations or to develop innovative architectures with the unique properties of Memristor. Future applications of Memistor and memristive systems are envisioned in both digital and analog domains. Major digital applications involve non-volatile solid-state memory, programmable logic and signal processing. Analog applications are related to programmable analog circuits, analog signal processing and neuromorphic applications .Some of these applications are briefly discussed here.

According to the International Technology Roadmap for Semiconductors (ITRS) report (ITRS 2007), it is predicted that, by 2019, 16nm half-pitch dynamic random access memory (DRAM) cells will provide a capacity of around 46GB/cm<sup>2</sup>. On this back ground nano scale memristors non volatile memory promise extremely high capacity of more than 110 and 460GB/cm<sup>2</sup> for 10 and 5nm half-pitch devices, respectively [9,10] and can replace the flash memory. Additionally, the devices ability to memorise a state without external biasing can significantly reduce the overall power consumption and would be potentially able to invent the **Boot Free Computers**.

Memristors have been proposed and demonstrated for different digital logic applications like flip flop [11] and logic gates [12]. Memristors could vastly improve a field programmable gate array, (FPGA) by replacing several specific transistors with a crossbar of memristors[13]. Application of discrete memristor as potentiometer enhances the performance of analog circuits with reduced area and added functionality [14]. Memristor-based programming of threshold, gain and frequency is experimented with a specially designed Memristor Emulator by Yuriy V. Pershin and Massimiliano Di Ventra [15]. Memristors have also been proposed in designing cellular neural networks (CNNs), recurrent neural networks, ultra wideband receivers, adaptive filters, oscillators, programmable threshold comparators, Schmitt triggers, difference amplifiers, in chaos circuits and also for biomimetic circuits [16].

The similarity of behavior in memristor and brain synapses is leading to develop real-time data analysis in neural network and neuromorphical applications. In such circuits the neurons are implemented with transistors, the axons with nano-wires in the crossbar and the synapses with the memristors [17]. It is expected that within a decade, memristors could potentially allow to properly emulate networks of neurons and synapses, instead of simply simulate.

## 6. Conclusion

Memristor is proving a promising advancement in fundamental circuit elements at nano scale. With its signature pinched hysteresis loop property it is exhibiting a non linear resistive switching. As on today the non linearity only at very low operating frequencies is an important limitation for its applications. Further, the internal state of an ideal memristor depends on the integral of the voltage or current over time. Therefore, its internal state, once programmed, would change significantly due to a dc component in current or applied voltage. Under this condition only perfect ac signals would be preferred to maintain the state of memristor thus narrowing its application areas. Though a lot of research on memristor device material is under taken, the optimization of material, Ron to Roff ratio, w/D ratio and doping motility  $\mu_v$  of an ideal Memristor is not yet achieved. In the non volatile memory application areas the problems of retention, endurance, switching speed, leakage mechanisms, yield, noise immunity are at infant research stage. In spite of all these constraints, the novelty of Memristor to combine computing, logic and memory on a single platform is a driving force behind the research and industry interest. The statement of Prof. Chua, "A time has come to re write the electrical technology books" is going to be a reality very soon.

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