B. N. BANDODKAR COLLEGE OF SCIENCE, THANE - 400 601.

FIRST TERM EXAMINATION OCT. - 2010

S. Y. B. Sc.

TIME : 2 Hrs.

SUBJECT : MATHEMATICS - I **MARKS :** 60

N. B. : 1. All questions are compulsory.

Q.1 a) State and prove Hausdroff property. [3]

b) Attempt ANY THREE of the following.

i) Define infimum and supremum of a set. Find infimum and supremum of the set A = (2, 5) [4]

ii) Prove that every subsequence of a convergent sequence is convergent. [4]

iii) Prove that every Cauchy sequence in $\Re$ is bounded. [4]

iv) If Σx_n converges then prove that $\lim_{x \to \infty} x_n = 0$. [4]

v) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ [4]

Q.2. a) State and prove Archimedean Property. [3]

b) Attempt ANY THREE of the following :

i) Prove that the union of arbitrary collection of open subsets in $\Re$ is open. [4]

ii) Prove that a subset of $\Re$ is closed if it contains all its limit points. [4]

iii) Define compact set. Give example of a set which is not compact. [4]

iv) Find limit points of A = (0, 1) [4]

v) State and prove Bernoulli's inequality. [4]

Q.3 a) Define a sequence of real numbers and limit of a sequence. [3]

b) Attempt ANY THREE of the following.

i) Prove that limit of a sequence, if exists, is unique. [4]

ii) Define monotonic increasing and monotonic decreasing sequence in $\Re$ with an example each. [4]

iii) State and prove Bolzano - Weierstrass theorem of sequences. [4]

iv) Find $\lim_{x \to \infty} a^n$, for $|a| < 1$. [4]

v) Show that the sequence $\left\langle \frac{4n^2 + 5}{3n^2 + 2} \right\rangle_{n=1}^{\infty}$

converges to $\frac{4}{3}$, using definition. [4]

Q.4 a) State and prove Ratio test. [3]

b) Attempt ANY THREE of the following :

i) Find the fourier series of $f(x) = x, x \in [-\pi, \pi]$ [4]

ii) Find the radius of convergence of the power series $\Sigma\, n!x^n$. [4]

iii) Prove that the series $\Sigma \frac{n^2 + 2n + 1}{n^3 + 2}$ is not convergent. [4]

iv) Discuss the convergence of the series $\Sigma \frac{\sqrt{n}}{2n + 3}$ [4]

v) State and prove Cauchy criterion for convergence of series. [4]